MA3402 HOMEWORK

Homeworks are intended as quick exercises to help solidify key ideas from the previous week. If you are spending a long time on them, stop and talk it over with someone.

1. For Thursday 7 September 2023

(1) Show that α^I for $I = (i_1 < \ldots < i_k)$ form a basis for $A_k(V)$. (If you get stuck, get inspiration from Propostion 10 (3.1) that α^i is a basis for $A_1(V)$.)

(2) Let $U \subset \mathbb{R}^n$ be an open subset of $\mathbb{R}^n$. Let $\omega \in \Omega^1(U)$, X a smooth vector field on U and $f \in C^\infty(U)$ a smooth map. Show that $\omega(fX) = f\omega(X)$.

(3) Let $\omega \in \Omega^2(U)$, $\tau \in \Omega^1(U)$. Let X, Y, Z be smooth vector fields on U. What is $(\omega \wedge \tau)(X, Y, Z)$?

(4) Let x^1, x^2, x^3, x^4 be the coordinates on $\mathbb{R}^4$, $p \in \mathbb{R}^4$. What is a basis for $A_3(T_p(\mathbb{R}^4))$?

(5) Let $\omega = f dx^I \in \Omega^k(U)$. Show that $d^2\omega = d(d(\omega))$ is zero. Conclude that $d^2(\omega) = 0$ for any $\omega \in \Omega^k(U)$.

(6) Make a cheat-sheet for yourself listing basis elements for $T_p(\mathbb{R}^n)$, $A_k(T_p(\mathbb{R}^n))$, $\Omega^k(U)$ for $k = 0, 1, 2$ and general k.