Norwegian University of Science and Technology
Department of Mathematical Sciences

MA3204 Homological Algebra Fall 2018

Exercise set 9

1 Calculate

a) $\operatorname{Ext}^n_{\mathbb{Z}}(\mathbb{Z}/2\mathbb{Z}, \mathbb{Q})$;

b) $\operatorname{Tor}_n^{\mathbb{Z}}(\mathbb{Q}, \mathbb{Z}/2\mathbb{Z})$;

c) $\operatorname{Tor}_n^{\mathbb{Z}}(\mathbb{Z}, \mathbb{Q})$.

2 Let $\mathbb{F}$ be a field and let $R = \mathbb{F}[X, Y]$, the polynomial ring in two variables.

a) Show that the *Koszul complex* is exact:

$$0 \to R \xrightarrow{\begin{pmatrix} Y \\ -X \end{pmatrix}} R \oplus R \xrightarrow{\begin{pmatrix} X & Y \end{pmatrix}} R \xrightarrow{f \mapsto f(0,0)} \mathbb{F} \to 0.$$

b) Calculate $\operatorname{Ext}^n_R(\mathbb{F}, \mathbb{F})$, where $\mathbb{F} \cong R/(X, Y)$ as an R-module.

3 An abelian group G is *divisible* if for any element $g \in G$ and any non-zero integer n, there is a $g' \in G$ such that $g = ng'$.

a) Show that any divisible group G is injective in $\mathbf{Ab}$.

b) Show that $\operatorname{Ext}^1_{\mathbb{Z}}(X, G) = 0$ for any abelian group X and any divisible group G.

4 Show that $\operatorname{Ext}^1_{\mathscr{A}}(A, B) = 0$ for all objects B in $\mathscr{A}$ if and only if A is projective.