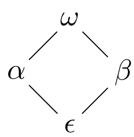
Norwegian University of Science and Technology
Department of Mathematical Sciences

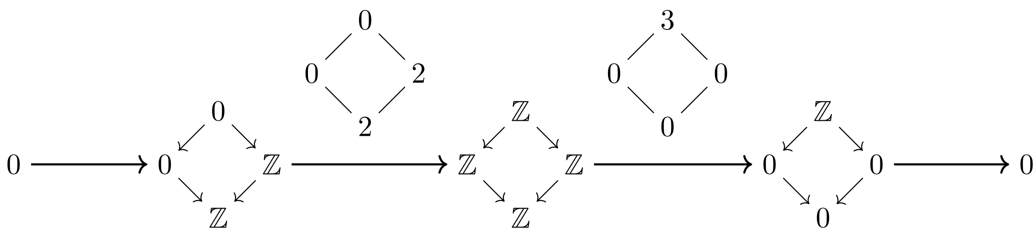
MA3204 Homological Algebra Fall 2018

Exercise set 7

1 Let X be the poset



Consider the complex in $\mathbf{C}(\text{presh}_{\mathbf{Ab}}X)$:



where "2" denotes multiplication by 2. Compute all the homologies of this complex.

2 Let f^* be a morphism of complexes. Show that if $\ker(f^*)$ and $\text{cok}(f^*)$ are *acyclic* (i.-e. exact complexes), then f^* is a *quasi-isomorphism* (i.-e. $H^n(f^*)$ is an isomorphism for all n).

3 Consider a morphism of complexes of the form:

$$\begin{array}{ccccccccccc}
\cdots & \longrightarrow & 0 & \longrightarrow & 0 & \longrightarrow & A^0 & \longrightarrow & 0 & \longrightarrow & \cdots \\
 & & \downarrow & & \downarrow & & \downarrow & & \downarrow & & \\
\cdots & \longrightarrow & 0 & \longrightarrow & B^{-1} & \longrightarrow & B^0 & \longrightarrow & 0 & \longrightarrow & \cdots
\end{array}$$

Show that if the morphism is a quasi-isomorphism, then there is a split short exact sequence

$$0 \to B^{-1} \to B^0 \to A^0 \to 0.$$

4 A complex A^* is called *split* if there are morphisms $h^n : A^n \to A^{n-1}$ such that $d^{n-1}h^n d^{n-1} = d^{n-1}$. In $\mathbf{C}(\mathbf{Mod}\,R)$:

a) Show that A^* is a split **exact** complex if and only if the identity map on A^* is null-homotopic.

b) Consider the complex $H^*(A^*)$ with 0 differentials:

$$\cdots \to H^{n-1}(A^*) \xrightarrow{0} H^n(A^*) \xrightarrow{0} H^{n+1}(A^*) \to \cdots$$

Show that A^* is split if and only if there is a homotopy equivalence between A^* and $H^*(A^*)$.