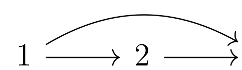
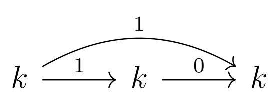
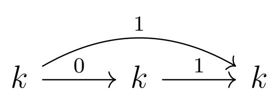
MA3203 - Exercise sheet 11

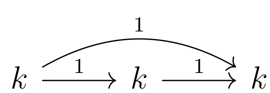
1. Let Λ be a ring and let $f : M \to N$ and $g : N \to L$ be essential epimorphisms of Λ-modules. Show that $g \circ f : M \to L$ is an essential epimorphism.

   Hint: Let $h\colon K \to M$ be a morphism, and assume $(g \circ f) \circ k$ is an epimorphism. First show that $g \circ k$ is an epimorphism, and then conclude that k must be an epimorphism.

2. [1, Exercise III.9ab] Find the radical and the top of each of the following representations of the quiver  3.

   (a) 

   (b) 

   (c) 

3. Let Λ be a left artinian ring, let $f\colon M \to N$ be an essential epimorphism between finitely generated modules, and let $h\colon M \to M$ be a morphism such that $f \circ h = f$. Show that h is an isomorphism.

   Hint: First show that h is an epimorphism. Then use that M has finite length.

4. Let Λ be a ring. We say that Λ is *local* if it has a unique maximal left ideal. Show that the following are equivalent

   (a) Λ is local

(b) The top of Λ is simple;

(c) The non-invertible elements of Λ form an ideal;

(d) Λ has a unique maximal right ideal.

Hint: Use what you know about the radical.

References

[1] M. Auslander, I. Reiten, and S. O. Smalø, *Representation Theory of Artin Algebras*, Cambridge Stud. Adv. Math. 36, Cambridge Univ. Press (1995).