

## MA3203 - Exercise sheet 5

Throughout  $k$  denotes a field.

1. [2, Problem 5.1] Find a composition series for the module  $\Lambda e_1$ , where  $\Lambda$  is the path algebra of each of the following quivers (with relations):

(a)  $\Lambda = k\Gamma$  for  $\Gamma = 1 \xrightarrow{\alpha} 2 \xrightarrow{\beta} 3$  .

(b)  $\Lambda = k\Gamma$  for  $\Gamma = 1 \xrightarrow{\alpha} 2 \begin{matrix} \xrightarrow{\beta} \\ \xrightarrow{\gamma} \end{matrix} 3$  .

(c)  $\Lambda = k\Gamma/(\beta\alpha)$  for  $\Gamma = 1 \xrightarrow{\alpha} 2 \begin{matrix} \xrightarrow{\beta} \\ \xrightarrow{\gamma} \end{matrix} 3$  .

(d)  $\Lambda = k\Gamma/(\beta\alpha, \delta^3)$  for  $\Gamma = 1 \xrightarrow{\alpha} 2 \begin{matrix} \xrightarrow{\beta} \\ \xrightarrow{\gamma} \end{matrix} 3 \begin{matrix} \curvearrowright \\ \curvearrowleft \end{matrix} \delta$  .

2. [1, Exercise I.5)] Let  $\Lambda$  be a ring, and let

$$0 \rightarrow L \xrightarrow{u} M \xrightarrow{r} N \rightarrow 0$$

be a short exact sequence of  $\Lambda$ -modules. Prove that  $u$  admits a retraction (a morphism  $v: M \rightarrow L$  such that  $v \circ u = 1_L$ ) if and only if  $r$  admits a section (a morphism  $s: N \rightarrow M$  such that  $r \circ s = 1_N$ ).

3. Here we consider modules of finite and infinite length.
- (a) Let  $\Lambda$  be a  $k$ -algebra. Show that if  $M$  is a  $\Lambda$ -module which has finite dimension as a  $k$ -vector space, then  $M$  has finite length.
  - (b) Let  $k[x]$  be the polynomial algebra in one variable. Show that  $k[x]$  does not have finite length as a module over itself.
  - (c) Given an example of a field  $k$  and an infinite-dimensional  $k$ -algebra which has finite length as a module over itself.

## References

- [1] I. Assem, D. Simson, and A. Skowroński, *Elements of the Representation Theory of Associative Algebras 1: Techniques of Representation Theory*, London Math. Soc. Stud. Texts 65, Cambridge Univ. Press (2006).
- [2] E. Hanson, 2021 MA3203 Problem Sheets, NTNU, [https://wiki.math.ntnu.no/ma3203/2021v/course\\_schedule](https://wiki.math.ntnu.no/ma3203/2021v/course_schedule).