

MA3203 - Exercise sheet 3

1. [1, Exercise III.8] Let $\Gamma = 1 \begin{array}{c} \xrightarrow{\alpha} \\ \xrightarrow{\beta} \end{array} 2$ be the Kronecker quiver. Define a representation H_λ of Γ by

$$H_\lambda = k \begin{array}{c} \xrightarrow{1} \\ \xrightarrow{\lambda} \end{array} k .$$

for every $\lambda \in k$. Show that H_λ is indecomposable and that $H_\lambda \cong H_\mu$ if and only if $\lambda = \mu$. This shows that $k\Gamma$ is not of finite representation type when k is infinite.

2. [4, Exercise 2.1] Let $\Gamma = 1 \xrightarrow{\alpha} 2 \xleftarrow{\beta} 3$ and let

$$V = k^2 \begin{array}{c} \xrightarrow{\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}} \\ \xrightarrow{\quad \quad} \end{array} k^2 \xleftarrow{\begin{array}{c} \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \\ \quad \quad \end{array}} k^2$$

. Write V as a direct sum of indecomposable representations of Γ .

3. Let $\Gamma = 1 \longrightarrow 2$ and let $V_1 \rightarrow V_2$ be a representation of Γ . Using Problem 2 b) in the previous exercise sheet, show that we can write $V_1 \rightarrow V_2$ as a direct sum

$$V_1 \rightarrow V_2 \cong (k \rightarrow 0)^{m_1} \oplus (0 \rightarrow k)^{m_2} \oplus (k \xrightarrow{1} k)^{m_3}$$

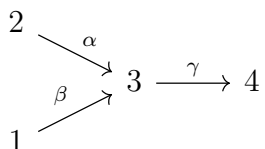
for *unique* integers m_1, m_2, m_3 . The uniqueness of this decomposition is a special case of the *Krull-Remak-Schmidt theorem*, which we will learn later in the course.

4. [2, Exercise 3.4] Let Γ be an arbitrary quiver and let V be a representation of Γ . Show that the following are equivalent.

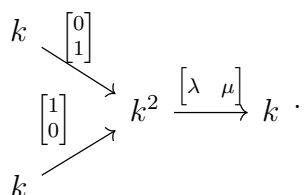
- (a) V is indecomposable.

- (b) If $\varphi : V \rightarrow V$ is idempotent ($\varphi^2 = \varphi$), then either $\varphi = 0$ (that is, $\varphi(i) = 0$ for all $i \in \Gamma_0$) or φ is an isomorphism¹.

5. [3, Exercise 2.5] Let Γ be the following quiver (of type D_4):



. For $\lambda, \mu \in k$, let $M(\lambda, \mu)$ be the following representation of Γ



- (a) Determine when $M(\lambda, \mu)$ and $M(\lambda', \mu')$ are isomorphic.
 (b) Determine for which values of λ and μ that $M(\lambda, \mu)$ is indecomposable.

References

- [1] I. Assem, D. Simson, and A. Skowroński, *Elements of the Representation Theory of Associative Algebras 1: Techniques of Representation Theory*, London Math. Soc. Stud. Texts 65, Cambridge Univ. Press (2006).
 [2] E. Hanson, 2021 MA3203 Problem Sheets, NTNU, https://wiki.math.ntnu.no/ma3203/2021v/course_schedule.
 [3] Ø. Solberg, 2017 MA3203 Problem Sheets, NTNU, <http://wiki.math.ntnu.no/ma3203/2017v/ovinger>.
 [4] L.-P. Thibault, 2019 MA3203 Problem Sheets, NTNU, http://wiki.math.ntnu.no/ma3203/2019v/lecture_plan.

¹This property is sometimes stated as “ $\text{End}(V)$ contains no nontrivial idempotents”.