MA3203 - Problem Set 11 (Projectives)

In all problems, K denotes a field, all representations are assumed to be finite dimensional representations over K, and all ideals are left ideals.

1. [1, Exercise 4.1] Let $Q = \; 1 \longrightarrow 2 \longrightarrow 3$. Find the projective cover and the kernel of the projective cover of each of the following representations of Q.

   (a) $K \xrightarrow{0} K \xrightarrow{0} K$

   (b) $K \xrightarrow{1} K \xrightarrow{0} K$

   (c) $K \xrightarrow{[1\ 0]} K^2 \xrightarrow{\begin{bmatrix}1\\1\end{bmatrix}} K$

2. [1, Exercise 4.2] Let $Q = \; 1 \longrightarrow 2 \rightrightarrows 3$. Find the projective cover and the kernel of the projective cover of each of the following representations of Q.

   (a) $K \xrightarrow{1} K \underset{\begin{bmatrix}0\\1\end{bmatrix}}{\overset{0}{\rightrightarrows}} K^2$

   (b) $K \xrightarrow{\begin{bmatrix}0\\1\end{bmatrix}} K^2 \underset{[1\ 1]}{\overset{[1\ 0]}{\rightrightarrows}} K$

   (c) $0 \longrightarrow K^2 \underset{\begin{bmatrix}1&0\\1&0\end{bmatrix}}{\overset{\begin{bmatrix}1&0\\1&0\end{bmatrix}}{\rightrightarrows}} K^2$

3. Let Λ be a ring and let M, N be Λ-modules. Show that $P \xrightarrow{f} M$ and $Q \xrightarrow{g} N$ are projective covers if and only if $P \oplus Q \xrightarrow{\begin{bmatrix}f&0\\0&g\end{bmatrix}} M \oplus N$ is a projective cover.

4. Find the projective covers and their kernels of the representations $K \xrightarrow{1} K \xleftarrow{1} K$ and $K \xleftarrow{1} K \xrightarrow{1} K$ of the quivers $1 \longrightarrow 2 \longleftarrow 3$ and $1 \longleftarrow 2 \longrightarrow 3$.

References

[1] Ø. Solberg, 2017 MA3203 Problem Sheets, NTNU, http://wiki.math.ntnu.no/ma3203/2017v/ovinger.