

MA3203 - Problem Set 11 (Projectives)

In all problems, K denotes a field, all representations are assumed to be finite dimensional representations over K , and all ideals are two-sided unless otherwise specified.

1. Find the projective covers and their kernels of the representations $K \xrightarrow{1} K \xleftarrow{1} K$ and $K \xleftarrow{1} K \xrightarrow{1} K$ of the quivers $1 \longrightarrow 2 \longleftarrow 3$ and $1 \longleftarrow 2 \longrightarrow 3$.

2. Let Λ be a ring and let M, N be Λ -modules. Show that $P \xrightarrow{f} M$ and $Q \xrightarrow{g} N$ are projective covers if and only if $P \oplus Q \xrightarrow{\begin{bmatrix} f & 0 \\ 0 & g \end{bmatrix}} M \oplus N$ is a projective cover.

3. [1, Exercise 4.1] Let $Q = 1 \longrightarrow 2 \longrightarrow 3$. Find the projective cover and the kernel of the projective cover of each of the following representations of Q .

(a) $K \xrightarrow{0} K \xrightarrow{0} K$

(b) $K \xrightarrow{1} K \xrightarrow{0} K$

(c) $K \xrightarrow{\begin{bmatrix} 1 & 0 \end{bmatrix}} K^2 \xrightarrow{\begin{bmatrix} 1 \\ 1 \end{bmatrix}} K$

4. [1, Exercise 4.2] Let $Q = 1 \longrightarrow 2 \rightrightarrows 3$. Find the projective cover and the kernel of the projective cover of each of the following representations of Q .

(a) $K \xrightarrow{1} K \xrightarrow{\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}} K^2$

(b) $K \xrightarrow{\begin{bmatrix} 0 \\ 1 \end{bmatrix}} K^2 \xrightarrow{\begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}} K$

(c) $0 \longrightarrow K^2 \xrightarrow{\begin{bmatrix} 1 & 0 \\ 1 & 0 \\ 1 & 0 \end{bmatrix}} K^2$

References

[1] Ø. Solberg, 2017 MA3203 Problem Sheets, NTNU, <http://wiki.math.ntnu.no/ma3203/2017v/ovinger>.