

## MA3203 - Problem Set 8 Answers and Hints

- 1) Let  $f \in \text{rad}K[x_1, \dots, x_n]$ . Then  $f$  is contained in the maximal ideal  $(x_1 - \lambda_1, \dots, x_n - \lambda_n)$  for any  $\lambda_1, \dots, \lambda_n \in K$ . This means  $x_i = \lambda_i$  is a zero of  $f$  for all  $i \in \{1, \dots, n\}$  and  $\lambda_i \in K$ . We conclude that  $f = 0$ .
- 3) Take  $Q$  to be a quiver with one vertex and one arrow.
- 5) Let  $x \in \text{rad}\Lambda$  and let  $y \in \Lambda'$ . Then there exists  $z \in \Lambda$  so that  $f(z) = y$ . We then have that  $1 - zx$  has some inverse  $w \in \Lambda$ . This means  $1 = f(w)(1 - yf(x))$ , and so  $1 - yf(x)$  is invertible (in  $\Lambda'$ ). We conclude that  $f(x) \in \text{rad}\Lambda'$ .