MA3203 - Problem Set 6 Answers and Hints

1a) Start with a composition series of N:

$$0 = N_0 \subseteq N_1 \subseteq \cdots \subseteq N_n = N.$$

Now recall that submodules of M/N correspond precisely to submodules of M containing N by the association $L \mapsto L/N$. Thus we can choose of composition series of M/N of the form:

$$0 = N/N = M_0/N \subseteq M_1/N \subseteq \cdots \subseteq M_m/N = M/N.$$

Putting these together gives a composition series

$$0 = N_0 \subseteq N_1 \subseteq \cdots \subseteq N_n = N = M_0 \subseteq M_1 \subseteq \cdots \subseteq M_m = M.$$

2a) Recall that $(M + N)/M \cong N/(M \cap N)$. This together with the theorem relating the lengths of modules appearing in an exact sequence will imply the result.

4d) Take $I = a\mathbb{Z}$ for any even integer a. Then $a/2 \in \ker \overline{2}$.