MA3203 - Problem Set 7 Answers and Hints

1a) From the previous problem set, we know N appears in a composition series for M. Since $\ell(M) = \ell(N)$, this will also be a composition series for M.

3a) Consider the commutative diagram corresponding to a morphism $M(1, \lambda) \to M(1, \lambda')$.

$$\begin{array}{ccc} K & \overset{\lambda}{\underset{1}{\rightrightarrows}} & K \\ \downarrow{\scriptstyle \mu} & & \downarrow{\scriptstyle \mu'} \\ K & \overset{\lambda'}{\underset{1}{\rightrightarrows}} & K \end{array} .$$

If $\lambda = \lambda'$, then this diagram commutes for any $\mu = \mu' \in K$. Otherwise, this diagram only commutes if $\mu = 0 = \mu'$.

3c) Write $K^n \cong K \oplus K^{n-1}$. Define $\phi : M(n-1, \lambda) \to M(n, \lambda)$ so that ϕ_i is the inclusion map $K^{n-1} \to K \oplus K^{n-1}$ for $i \in \{1, 2\}$. Likewise, define $\psi : M(n, \lambda) \to M(1, \lambda)$ so that ψ_i is the projection map $K \oplus K^{n-1} \to K$ for $i \in \{1, 2\}$. These maps give the desired exact sequence.