MA3203 - Problem Set Hints and Answers

1a) We first have $\Lambda e_1 = K \xrightarrow{1} K \xrightarrow{1} K$. Now define

$$\begin{aligned} M_1 &= 0 \to 0 \to K \\ M_2 &= 0 \to K \xrightarrow{1} K. \end{aligned}$$

Then there is a composition series $0 \subseteq M_1 \subseteq M_2 \subseteq \Lambda e_1$, where 0 denotes the 0 representation.

1c) We first have $\Lambda e_1 = K \xrightarrow{1} K \overset{\begin{bmatrix}1\\0\end{bmatrix}}{\underset{\begin{bmatrix}0\\1\end{bmatrix}}{\rightrightarrows}} K^2$. Now define

$$\begin{aligned} M_1 &= 0 \longrightarrow 0 \rightrightarrows K \\ M_2 &= 0 \longrightarrow 0 \rightrightarrows K^2 \\ M_3 &= 0 \longrightarrow K \overset{\begin{bmatrix}1\\0\end{bmatrix}}{\underset{\begin{bmatrix}0\\1\end{bmatrix}}{\rightrightarrows}} K^2. \end{aligned}$$

Then there is a composition series $0 \subseteq M_1 \subseteq M_2 \subseteq M_3 \subseteq \Lambda e_1$, where 0 denotes the 0 representation.

2c) If K is an infinite field, both have infinitely many. A harder (but interesting!) problem is to compute the number of composition series when K is the field with 2 elements.

3a) The only subrepresentations of this representation are $0 \to K \xrightarrow{1} K \leftarrow 0$ and $0 \to 0 \to K \leftarrow 0$ (and the 0 representation). These can only be arranged into a composition series in one way.

3b) Let $M = 0 \to K \xrightarrow{1} K \xleftarrow{1} K$. The subrepresentations of M are then:

$$\begin{aligned} N &= 0 \to K \xrightarrow{1} K \leftarrow 0 \\ N' &= 0 \to 0 \to K \xleftarrow{1} K \\ S_3 &= 0 \to 0 \to K \leftarrow 0 \end{aligned}$$

and the 0 representation. There are then two composition series:

$$0 \subseteq S_3 \subseteq N \subseteq M, \qquad 0 \subseteq S_3 \subseteq N' \subseteq M.$$

3d) Let

$$0 = M_0 \subseteq M_1 \subseteq \cdots \subseteq M_m = M, \qquad 0 = N_0 \subseteq N_1 \subseteq \cdots \subseteq N_n = N$$

be composition series for M and N. Then there is a composition series

$$0 = M_0 \subseteq M_1 \subseteq \cdots \subseteq M_m = M = M \oplus N_0 \subseteq M \oplus N_1 \subseteq \cdots \subseteq M \oplus N_n = M \oplus N.$$

By switching each M and N, we obtain a second composition series, meaning $M \oplus N$ is not uniserial.