MA3203 - Problem Set 4 (Quivers with Relations)

1c) One isomorphism is the linear map sending e_1 to the identity matrix, α to the matrix with 1 in position $(3, 2)$, and β to the matrix with 1 in position $(2, 1)$. *Note: e_1, α, and β are technically referring to equivalence classes in the quotient algebra rather than elements in KQ.*

2a) *Hint: We have essentially eliminated elements corresponding to paths of length larger than m. What can we say about the number of paths left?*

3a) For example, we have

$$\Lambda e_2 = \ K \underset{[1\ 0]}{\overset{\begin{bmatrix}0\\1\end{bmatrix}}{\rightleftarrows}} K^2 \underset{\begin{bmatrix}0\\-1\end{bmatrix}}{\overset{[1\ 0]}{\rightleftarrows}} K \ .$$

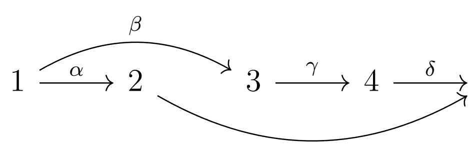
3b) There is a morphism $\phi : \Lambda e_2 \to \Lambda e_2$ with $\phi(1) = 0 = \phi(3)$ and $\phi(2) = \begin{bmatrix}0 & 0\\1 & 0\end{bmatrix}$.

3c) Let ϕ be the morphism found in 3b. It can be shown that $\mathrm{Hom}_\Lambda(\Lambda e_2, \Lambda e_2)$ is a 2-dimensional vector space with basis $\{1_{\Lambda e_2}, \phi\}$. Then for $\lambda_1, \lambda_2 \in K$, we have

$$(\lambda_1 \cdot 1_{\Lambda e_2} + \lambda_2 \cdot \phi) \circ (\lambda_1 \cdot 1_{\lambda e_2} + \lambda_2 \cdot \phi) = \lambda_1^2 \cdot 1_{\Lambda e_2} + 2\lambda_1\lambda_2 \cdot \phi.$$

Thus $\lambda_1 \cdot 1_{\Lambda e_2} + \lambda_2 \cdot \phi$ is idempotent if and only if $\lambda_1 = \lambda_1^2$ (so that λ_1 must be 0 or 1_K) and $2\lambda_1\lambda_2 = \lambda_2$. If $\lambda_1 = 0$, this means $\lambda_2 = 0$ and so $(\lambda_1 \cdot 1_{\Lambda e_2} + \lambda_2 \cdot \phi) = 0$. Otherwise, this means $2\lambda_2 = \lambda_2$ (so that λ_2 must be 0) and $(\lambda_1 \cdot 1_{\Lambda e_2} + \lambda_2 \cdot \phi) = 1_{\Lambda e_2}$. We conclude that there are no nontrivial idempotent endomorphisms $\Lambda e_2 \to \Lambda e_2$, and thus Λe_2 is indecomposable.

4a) Let $Q =$



. The the matrix algebra is isomorphic to $KQ/(\epsilon\alpha - \delta\gamma\beta)$.