MA3203 - Problem Set 2 Answers and Hints

1a) The morphisms $(V, f) \to (V', f')$ are given as follows, where $a, b, c \in K$ are arbitrary.

$$\begin{array}{ccccc} K^2 & \xrightarrow{[1\ 1]} & K & \xrightarrow{\begin{bmatrix}0\\1\end{bmatrix}} & K^2 \\ {\scriptstyle [a\ a]}\Big\downarrow & & \Big\downarrow{\scriptstyle \begin{bmatrix}a\\0\end{bmatrix}} & & \Big\downarrow{\scriptstyle \begin{bmatrix}c & a\\ b & 0\end{bmatrix}} \\ K & \xrightarrow{\begin{bmatrix}1\\0\end{bmatrix}} & K^2 & \xrightarrow{\begin{bmatrix}1 & 0\\0 & 0\end{bmatrix}} & K^2 \end{array}$$

1b) The subrepresentations are:

$$K \xrightarrow{1} K \xrightarrow{1} K, \qquad 0 \to K \xrightarrow{1} K, \qquad 0 \to 0 \to K, \qquad 0 \to 0 \to 0$$

2a) Consider the following diagram:

$$\begin{array}{ccc} K & \overset{\mu}{\underset{\lambda}{\rightrightarrows}} & K \\ \Big\downarrow{\scriptstyle \nu} & & \Big\downarrow{\scriptstyle \nu'} \\ K & \overset{\mu'}{\underset{\lambda'}{\rightrightarrows}} & K \end{array} .$$

This corresponds to a morphism $M(\lambda, \mu) \to M(\lambda', \mu')$ if and only if $\mu\nu' = \mu'\nu$ and $\lambda\nu' = \nu'\lambda$. In particular, if $\lambda'\lambda^{-1} \neq \mu'\mu^{-1}$, the we must have $\nu = 0 = \nu'$. Therefore in this case, there is no isomorphism $M(\lambda, \mu) \to M(\lambda', \mu')$. Otherwise, we can choose $\nu = 1$ and $\nu' = \mu'\mu^{-1} = \lambda'\lambda^{-1}$. In this case, the diagram describes an isomorphism $M(\lambda, \mu) \to M(\lambda', \mu')$.

2b) The inverse is given by $\nu \mapsto M(\nu, 1)$. This is indeed an inverse since $M(\lambda\mu^{-1}, 1) \cong M(\lambda, \mu)$ be part (a).

3) We can take $n = 1$ and consider $M(1, 1)$ and $M(1, -1)$.

4a) Let $\alpha : i \to j$ be an arrow in Γ. Then the following diagram must commute:

$$\begin{array}{ccc} V(i) & \xrightarrow{f_\alpha} & V(j) \\ \Big\downarrow{\scriptstyle \phi_i} & & \Big\downarrow{\scriptstyle \phi_j} \\ V'(i) & \xrightarrow{f'_\alpha} & V'(j) \end{array} .$$

In particular, if $x \in \ker(\phi_i)$, then $0 = f'_\alpha \circ \phi_i(x) = \phi_j \circ f_\alpha(x)$. This means $\phi_i(x) \in \ker(\phi_j)$, as desired.

4b) In the diagram for (2a), suppose $a = 1$, $b = 0$, and $c = -1$. Then we have

$$\ker(\phi) = \; K\binom{1}{-1} \longrightarrow 0 \longrightarrow K\binom{1}{1} \;\cong K \to 0 \to K.$$