MA3203 - Problem Set 3 Answers and Hints

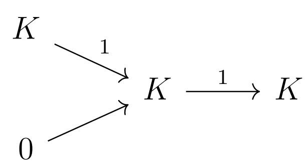
2) For $i \in \Gamma_0$, let S_i be the representation with K at vertex i and 0 everywhere else. Then $V \cong S_1 \oplus S_2 \oplus S_3 \oplus (K \xrightarrow{1} K \xrightarrow{1} K)$.

3a) $V(\lambda, \mu)$ is indecomposable if and only if λ and μ are both nonzero.

3b) $V(\lambda, \mu) \cong V(\lambda', \mu')$ if and only if any of the following hold:

1. $\lambda = 0 = \lambda'$ and μ and μ' are either both zero or both nonzero.
2. $\mu = 0 = \mu'$ and λ and λ' are either both zero or both nonzero.
3. All of λ, λ', μ, and μ' are nonzero.

3c) For example, the representation corresponding to Λe_2 is



3d) Each subrepresentation (other than the zero representation) is also of the form Λe_j for some $j \in \Gamma_0$.

4) ($\Rightarrow$) Let $\phi : V \to V$ be idempotent. For each $i \in \Gamma_0$, decompose $V(i) \cong \ker(\phi_i) \oplus \mathrm{Im}\phi_i$. Let $\alpha : i \to j$ be an arrow in Γ_1, and suppose f is the linear map $V(i) \to V(j)$ corresponding to α. Show that if $x \in \ker(\phi_i)$ then $f(x) \in \ker(\phi_j)$ and if $x \in \mathrm{Im}\phi_i$ then $f(x) \in \mathrm{Im}(\phi_j)$. This will give a direct sum decomposition $V \cong \ker(\phi) \oplus \mathrm{Im}(\phi)$. Since V is indecomposable, this means either $\phi = 0$ or $\phi = 1_V$.

($\Leftarrow$) Let $V \cong V' \oplus V''$ be an arbitrary direct sum decomposition. Let $\iota : V' \to V' \oplus V''$ be the inclusion map and $\pi : V' \oplus V'' \to V'$ the projection map. It follows that $\iota \circ \pi$ is an idempotent $V' \oplus V'' \to V' \oplus V''$. Thus either $\iota \circ \pi = 0$, in which case $V' = 0$, or $\iota = 0$, in which case $V'' = 0$.