MA3203 - Problem Set 3 (Representations of Quivers)

In all problems, K denotes a field and all representations are representations over K.

1. Let $\Gamma = \; 1 \overset{\alpha}{\underset{\beta}{\rightrightarrows}} 2$. For all $\lambda \in K \setminus \{0\}$, let $M(\lambda) = \; K \overset{1}{\underset{\lambda}{\rightrightarrows}} K$. Show that $M(\lambda)$ is indecomposable.

2. [2, Exercise 2.1] Let $\Gamma = \; 1 \xrightarrow{\alpha} 2 \xrightarrow{\beta} 3$ and let $V = K^2 \xrightarrow{\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}} K^2 \xrightarrow{\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}} K^2$. Write V as a direct sum of indecomposable representations.

3. Consider the following quiver (of type D_4):

$$\Gamma = \quad \begin{array}{ccccc} 2 & & & & \\ & \overset{\alpha}{\searrow} & & & \\ & & 3 & \xrightarrow{\gamma} & 4 \\ & \underset{\beta}{\nearrow} & & & \\ 1 & & & & \end{array}$$

   (a) [1, Exercise III.7] For $\lambda, \mu \in K$, let $V(\lambda, \mu)$ be the following representation of Γ:

$$V(\lambda, \mu) = \quad \begin{array}{ccccc} K^2 & & & & \\ & \overset{\begin{bmatrix} 0 \\ 1 \end{bmatrix}}{\searrow} & & & \\ & & K^2 & \xrightarrow{\begin{bmatrix} \lambda & \mu \end{bmatrix}} & K \\ & \overset{\begin{bmatrix} 1 \\ 0 \end{bmatrix}}{\nearrow} & & & \\ K & & & & \end{array}$$

   of Γ. For what values of λ and μ is $V(\lambda, \mu)$ indecomposable?

   (b) [1, Exercise III.7] When are $V(\lambda, \mu)$ and $V(\lambda', \mu')$ isomorphic?

   (c) Find the representation corresponding to the module Λe_i for each $i \in \Gamma_0$.

   (d) Find all of the subrepresentations of each of the representations you found in part (c). Do you notice anything particular about them?

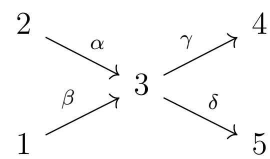
4. Let Γ be an arbitrary quiver and let V be a representation of Γ. Show that the following are equivalent.

   (a) V is indecomposable.

   (b) If $\varphi : V \to V$ is idempotent ($\varphi^2 = \varphi$), then either $\varphi = 0$ (that is, $\varphi(i) = 0$ for all $i \in \Gamma_0$) or φ is an isomorphism[1].

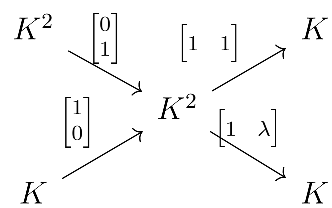
5. Consider the following quiver:

$\Gamma =$



For $\lambda \in K \setminus \{0\}$, let $V(\lambda)$ be the following representation of Γ:

$V(\lambda) =$



   (a) Show that $V(\lambda) \cong V(\lambda')$ if and only if $\lambda = \lambda'$.

   (b) Show that $V(\lambda)$ is indecomposable.

   (c) Conclude that the path algebra $K\Gamma$ is not of finite representation type.

References

[1] M. Auslander, I. Reiten, and S. O. Smalø, *Representation Theory of Artin Algebras*, Cambridge Stud. Adv. Math. 36, Cambridge Univ. Press (1995).

[2] L.-P. Thibault, 2019 MA3203 Problem Sheets, NTNU, http://wiki.math.ntnu.no/ma3203/2019v/lecture_plan.

[1] This property is sometimes stated as "End(V) contains no nontrivial idempotents".