Norwegian University of Science and Technology
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MA3203
Ring theory
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Exercise set 4

1 Let Q be the quiver $1 \longrightarrow 2 \longrightarrow 3 \longleftarrow 4 \longrightarrow 5 \longleftarrow 6$.

Find a composition series for the module of dimension vector (0,1,1,1,1,1).

Hint: You have probably noticed that if you have an arrow $\alpha : i \to j$ represented by $f_\alpha : k \to k$, then a representation with $f_\alpha : k \to 0$ cannot be a subrepresentation. Thus, to get you started, you should probably take S_i as your first simple submodule for some vertex i which is a sink (a sink is a vertex such that all edges connected to it point towards it).

2 Let M be a module of finite length and N and L be submodules. Show that

$$l(N + L) + l(N \cap L) = l(N) + l(L).$$

Hint: You will obtain the formula by considering two short exact sequences.

Remark: This generalizes the formula for vector spaces:

$$\dim(V + W) + \dim(V \cap W) = \dim V + \dim W.$$

3 Let M be a kQ-module, where Q is a finite acyclic connected quiver. Show that the number of appearances of the simple module S_i as a composition factor of M is equal to $\dim_k e_i M$.

Hint: Given a composition series for M:

$$0 = M_0 \subset \cdots \subset M_l = M,$$

Consider the short exact sequences of vector spaces

$$0 \to e_i M_{t-1} \to e_i M_t \to e_i M_t / M_{t-1} \to 0,$$

for $t = 1, \ldots, l$. What is the dimension of $e_i M_t$ in terms of the other two? You eventually want the dimension of $e_i M_l$.