Grading document for MA3202 June 3rd 2024

For the final grade the official scale of NTNU was used, that is

- A: 89–100 points
- B: 77–88 points
- C: 65–76 points
- D: 53–64 points
- E: 41–52 points
- F: 0–40 points

For each part of a problem, any solution which was correct would get full points. If a solution was partly correct, points would be given depending on the correct parts as per the following guide. Calculation mistakes that do not affect the arguments involved do not subtract points. Discretion is exercised when awarding points for all problems.

Problem 1

(a)
   (i) Correct solution: 10 points.
   (ii) Showing correctly that units in $F[x]$ are nonzero constants: +9 points.
   (iii) Showing correctly that nonzero constants in $F[x]$ are units: +1 point.

(b)
   (i) Correct solution: 10 points.
   (ii) Showing correctly that $\deg(f(x)) \geq 1$: +5 points.
   (iii) Noticing that showing that $f(x)$ has a root in $\mathbb{Z}_p$ for a prime p implies $\deg(f) \leq 1$: +2 points.
   (iv) Showing correctly that $f(x)$ has a root in $\mathbb{Z}_p$ for a prime p: +3 points.

Problem 2

(a)
   (i) Correct solution: 10 points.
   (ii) Showing correctly that the extension $\mathbb{Q} \subseteq E$ is Galois: +2 points.
   (iii) Showing correctly that $[\mathbb{Q}(\sqrt{p}) : \mathbb{Q}] = 2$: +2 points.
   (iv) Showing correctly that $[\mathbb{Q}(\sqrt{p}, \sqrt{q}) : \mathbb{Q}(\sqrt{p})] = 2$: +2 points.
   (v) Showing correctly that $|\mathrm{Gal}(E/\mathbb{Q})| = 4$: +2 points.
   (vi) Showing correctly that $\mathrm{Gal}(E/\mathbb{Q}) \cong \mathbb{Z}_2 \times \mathbb{Z}_2$: +2 points.

(b)
   (i) Correct solution: 10 points.
   (ii) Showing correctly that there exist 3 intermediate fields: +5 points.
   (iii) Describing correctly the 3 intermediate fields: +1 point.
   (iv) Showing correctly that the three intermediate fields are $\mathbb{Q}(\sqrt{p})$, $\mathbb{Q}(\sqrt{q})$, $\mathbb{Q}(\sqrt{pq})$: +4 points.

Problem 3

(a) (i) Correct solution: 7 points.
  (ii) Showing correctly that $[\mathbb{Q}(\alpha_1, \alpha_2, \alpha_3) : \mathbb{Q}] \geq [\mathbb{Q}(\alpha_1) : \mathbb{Q}]$ where $\alpha_1, \alpha_2, \alpha_3$ are the roots of $f(x)$: +3 points.
  (iii) Showing correctly that $[\mathbb{Q}(\alpha_1) : \mathbb{Q}] = 3$: +2 points.
  (iv) Showing correctly that $|G| = [\mathbb{Q}(\alpha_1, \alpha_2, \alpha_3) : \mathbb{Q}]$: +2 points.

(b) (i) Correct solution: 6 points.
  (ii) Showing correctly that $G \cong \mathbb{Z}_n$ for some integer $n \in \mathbb{Z}$: +1 point.
  (iii) Showing correctly that $3 \mid n$: +1 point.
  (iv) Showing correctly that $n = 3$: +4 points.

(c) (i) Correct solution: 7 points.
  (ii) Showing correctly that if $z \in \mathbb{C}$ is a root of $f(x)$ then $\overline{z}$ is also a root of $f(x)$: +4 points.
  (iii) Showing correctly that all of the roots of $f(x)$ are real: +3 points.

Problem 4

(a) (i) Correct solution: 7 points. **Unfortunately there is an ambiguity in the first part of the question: it can be interpreted as either showing that $E = \mathrm{GF}(7^2)$ or as E has 49 elements. Both interpretations are considered correct when grading.**
  (ii) Showing correctly that $E = \mathrm{GF}(7^2)$ is a field with 49 elements: +4 points.
  (iii) Showing correctly that $[E : F] = 2$: +3 points.

(b) (i) Correct solution: 7 points.
  (ii) Showing correctly that if $g(x)$ divides $f(x)$ then $\deg(g) = 1$ or 2: +3 points.
  (iii) Showing correctly that if $\deg(g) = 1$, then $g(x)$ divides $f(x)$:+2 points.
  (iv) Showing correctly that if $\deg(g) = 2$, then $g(x)$ divides $f(x)$: +2 points.

(c) (i) Correct solution: 6 points.
  (ii) Noticing that $f(x)$ is a product of irreducible monic polynomials of degrees 1 or 2: +1 points.
  (iii) Noticing that all irreducible polynomials of degrees 1 or 2 appear as factors of $f(x)$: +2 point.
  (iv) Counting correctly the irreducible polynomials of degree 2 that divide $f(x)$: +3 points.
  (v) Counting correctly all monic polynomials of degree 2: +1 point.
  (vi) Counting correctly all monic reducible polynomials of degree 2: +5 points.

Problem 5

(a) (i) Correct solution: 5 points.
  (ii) Noticing that $\Phi_7(x)$ is irreducible if and only if $\Phi_7(x+1)$ is irreducible: +1 point.
  (iii) Computing correctly $\Phi_7(x+1)$: +2 points.
  (iv) Showing correctly that $\Phi_7(x+1)$ is irreducible: +2 points.

(b) (i) Correct solution: 5 points.
  (ii) Showing correctly that $\Phi_7(x)$ splits in $\mathbb{Q}(\omega)$: +3 points.
  (iii) Showing correctly that $\mathbb{Q}(\omega)$ is the smallest field in which $\Phi_7(x)$ splits: +2 points.

(c) (i) Correct solution: 5 points.

(ii) Showing correctly that $\mathrm{Gal}(\mathbb{Q}(\omega)/\mathbb{Q})$ has 6 elements: +1 point.

(iii) Defining a map f between $\mathrm{Gal}(\mathbb{Q}(\omega)/\mathbb{Q})$ and $\Phi_7(x)$: +1 point.

(iv) Showing that the map f is a group homomorphism: +1 point.

(v) Showing that the map f is injective: +1 point.

(vi) Showing that the map f is surjective: +1 point.

(d) (i) Correct solution: 5 points.

(ii) Computing correctly the minimal polynomial of ρ: +2 points.

(iii) Computing correctly the order of the group $\mathrm{Gal}(\mathbb{Q}(\omega)/\mathbb{Q}(\rho))$: +1 point.

(iv) Showing correctly that $\mathrm{Gal}(\mathbb{Q}(\omega)/\mathbb{Q}(\rho)) \cong \mathbb{Z}_3$: +2 points.