Grading document for MA3202 June 3rd 2023

For the final grade the official scale of NTNU was used, that is

- A: 89–100 points
- B: 77–88 points
- C: 65–76 points
- D: 53–64 points
- E: 41–52 points
- F: 0–40 points

For each part of a problem, any solution which was correct would get full points. If a solution was partly correct, points would be given depending on the correct parts as per the following guide. Calculation mistakes that do not affect the arguments involved do not subtract points. Discretion is exercised when awarding points for all problems.

Problem 1

(a) (i) Correct solution: 7 points.

(ii) Showing that the polynomial has no roots: +3 points.

(iii) Mentioning that the degree is 3 and since there are no roots the polynomials is irreducible: +4 points.

(iv) Trying to show explicitly that $p(x)$ can't be written as a product of two polynomials of smaller degrees but not managing to complete the proof correctly: +4 points.

(b) (i) Correct solution: 7 points.

(ii) Showing that $\mathbb{Z}_5(\alpha)$ has 5^3 elements: +3 point.

(iii) Showing that $\beta^{5^3} = \beta$ for all $\beta \in \mathbb{Z}_5(\alpha) \setminus \{0\}$: +3 points.

(iv) Showing or mentioning that $0^{5^3} = 0$: +1 point.

(v) Using the general fact that for all $x \in \mathrm{GF}(p^n)$ we have $x^{p^n} = x$: +2 points.

(c) (i) Correct solution: 6 points.

(ii) Showing that $\phi^3 = \mathrm{id}$: +2 points.

(iii) Computing wrongly that $\phi^n = \mathrm{id}$ for some $n \neq 3$: +1 point.

(iv) Mentioning that we have to check that ϕ cannot have order 1 or 2: + 1 point.

(v) Showing that ϕ cannot have order 1 or 2: +2 points.

Problem 2

(a) (i) Correct solution: 7 points.
  (ii) Writing the roots of $p(x)$ correctly: +2 points.
  (iii) Showing that the roots of $p(x)$ belong to $\mathbb{Q}(\sqrt[3]{2}, i\sqrt{3})$: +5 points.

(b) (i) Correct solution: 7 points.
  (ii) Proving that $x^3 - 2$ is irreducible: +3 points.
  (iii) Proving that $x^3 - 2$ has one but not all roots in $\mathbb{Q}(\sqrt[3]{2})$: +3 points.
  (iv) Mentioning that irreducibility of $x^3 - 2$ is required to conclude that $\mathbb{Q} \subseteq \mathbb{Q}(\sqrt[3]{2})$ is not normal: +1 point.

(c) (i) Correct solution: 6 points.
  (ii) Using the formula for a sequence of field extensions: +1 point.
  (iii) Computing correctly the degree of each intermediate extension: +1 points for each extension.
  (iv) Describing the elements of the Galois group correctly: +2 points.
  (v) Comparing the structure of the Galois group with the groups $\mathbb{Z}_6$ and S_3 correctly: +1 points.
  (vi) Using the FTGT correctly to argue why the Galois group can not have normal subgroups: +3 points.

Problem 3

(a) (i) Correct solution: 10 points.
  (ii) Showing the result when $\deg(p_\alpha(x)) = 1$: +2 points.
  (iii) Showing that $\alpha \notin K$: +2 points.
  (iv) Observing that one also needs to show that higher powers of α are not in K: +3 points.

(b) (i) Correct solution: 10 points.
  (ii) Showing that $n[F(\beta) : F(\alpha) \cap F(\beta)] = m[F(\alpha) : F(\alpha) \cap F(\beta)]$ or $n[F(\alpha, \beta) : F(\alpha)] = m[F(\alpha, \beta) : F(\beta)]$: +1 point.
  (iii) Correctly mentioning how to apply part (a): +2 points.
  (iv) Showing that $[F(\alpha, \beta) : F] = nm$ correctly: +3 points.

Problem 4

(a) (i) Correct solution: 7 points.
  (ii) Showing that $\omega, \omega^2, \ldots, \omega^{16}$ are roots of $\Phi_{17}(x)$: +3 points.
  (iii) Showing that $\omega, \omega^2, \ldots, \omega^{16}$ are distinct: +3 points.
  (iv) Observing that $\mathbb{Q}(\omega, \omega^2, \ldots, \omega^{16}) = \mathbb{Q}(\omega)$: +1 point.

(b) (i) Correct solution: 7 points.
  (ii) Describing $\text{Gal}(\mathbb{Q}(\omega)/\mathbb{Q})$: +3 points.
  (iii) Defining a map between $\text{Gal}(\mathbb{Q}(\omega)/\mathbb{Q})$ and $\mathbb{Z}_{17}^\times$: +1 points.
  (iv) Showing that the map defined in (iii) is a group homomorphism: +2 points.
  (v) Showing that the map defined in (iii) is a bijection: +1 point.

(c) (i) Correct solution: 6 points.
  (ii) Obtaining a sequence of subgroups of $\text{Gal}(\mathbb{Q}(\omega)/\mathbb{Q})$ such that the quotient of two consecutive terms has order 2: +2 points.
  (iii) Translating the sequence obtained in (ii) to a sequence of field inclusions: +2 points.
  (iv) Showing that the sequence of field inclusions obtained in (iii) has the correct degrees: +2 points.

Problem 5

(a)
  (i) Correct solution: 5 points.
  (ii) Showing that ± 1 are units: +1 point.
  (iii) Showing that units have to be constants: +2 points.
  (iv) Showing that units have to be ± 1: +2 points.

(b)
  (i) Correct solution: 5 points.
  (ii) Showing that $f(x)$ irreducible in R implies $f(x) = \pm p$ for a prime number $p \in \mathbb{Z}$: +3 points.
  (iii) Showing that $f(x) = \pm p$ for a prime number $p \in \mathbb{Z}$ is irreducible in R: +2 points.

(c)
  (i) Correct solution: 5 points.
  (ii) Showing that if $f(x)$ is irreducible in $\mathbb{Q}[x]$ and $f(0) = \pm 1$, then $f(x)$ is irreducible in R: +1 point.
  (iii) Showing that if $f(x)$ is irreducible in R, then $f(x)$ is irreducible in $\mathbb{Q}[x]$: +1 point.
  (iv) Showing that if $f(x)$ is irreducible in R, then $f(0) \in \{-1, 0, 1\}$: +2 points.
  (v) Showing that if $f(x)$ is irreducible in R, then $f(0) \neq 0$: +1 point.

(d)
  (i) Correct solution: 5 points.
  (ii) Showing that x is not irreducible: +1 point.
  (iii) Showing that there are many different factorizations of x but not mentioning that these are factorizations to a product of irreducibles: +2 points.
  (iv) Showing that x does not have a factorization into exactly two irreducibles: +3 points.