Norwegian University of Science and Technology
Department of Mathematical Sciences

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Exam in MA3202 Galoisteori

English
Saturday 19. may 2012
Time: 09.00 - 13:00
Permitted aids: No printed or handwritten aids permitted
Results: 15. june 2012

Problem 1

a) Show that $2+i$ is irreducible in $\mathbf{Z}[i] = \{m + ni \mid m, n \in \mathbf{Z}\}$, and that 5 is reducible in $\mathbf{Z}[i]$.

b) Which of the numbers $11, 13$ and 19 are irreducible in $\mathbf{Z}[i]$? Give reasons.

Problem 2

Let $\omega \in \mathbf{C}$ be a primitive 13th root of unity, and let $E = \mathbf{Q}(\omega)$.

a) Show that E is a normal extension of $\mathbf{Q}$, and determine the cyclic Galois group $G = G(E|\mathbf{Q})$.

b) How many proper intermediate fields K, $\mathbf{Q} \underset{\neq}{\subset} K \underset{\neq}{\subset} E$, are there? Give reasons.

Problem 3

Let $f(x) = x^7 - 2 \in \mathbf{Q}[x]$, and let E be the splitting field of $f(x)$ over $\mathbf{Q}$.

a) Determine the order of the Galois group $G = G(E|\mathbf{Q})$.

b) Show that there exists a non-normal subgroup H of G.

c) Show that there exists a normal subgroup N of G such that G/N is abelian.

Problem 4

Let $E = \mathbb{Q}(\sqrt{2} - 3\sqrt{3})$.

a) Show that E is a normal extension of $\mathbb{Q}$, and determine $G = G(E|\mathbb{Q})$.

b) List all the intermediate fields $K, \mathbb{Q} \subseteq K \subseteq E$.

Problem 5

We denote by $GF(p^n)$ the finite field with p^n elements, where p is a prime.

a) Let $f(x)$ be an irreducible polynomial of degree 36 over $GF(5)$. By referring to relevant theorems show that $f(x)$ divides the polynomial

$$x^{5^{36}} - x$$

in $GF(5)[x]$, and that $f(x)$ has distinct roots.

b) Let E be the splitting field of $f(x)$ over $GF(5)$. Exhibit by a diagram the inclusion of the intermediate fields $K, GF(5) \subseteq K \subseteq E$.