

Grading document for MA3201 December 12th 2023

For the final grade the official scale of NTNU was used, that is

- A: 89–100 points
- B: 77–88 points
- C: 65–76 points
- D: 53–64 points
- E: 41–52 points
- F: 0–40 points

For each part of a problem, any solution which was correct would get full points. If a solution was partly correct, points would be given depending on the correct parts as per the following guide. Calculation mistakes that do not affect the arguments involved do not subtract points. Discretion is exercised when awarding points for all problems.

Problem 1

- (a) (i) Correct solution: 15 points.
(ii) Computing the Smith normal form with exactly one unallowed operation: +5 points.
- (b) (i) Correct solution: 5 points.
(ii) Computing the rational canonical form correctly up to mistake in application of the definition: +4 points.
(iii) Computing the rational canonical form without using monic polynomials: +3 points.
(iv) Mentioning only the general way of computing the rational canonical form and not the specific form in the example: +2 points.
- (c) (i) Correct solution: 5 points.
(ii) Computing the Jordan canonical form correctly up to mistake in application of the definition: +4 points.
(iii) Computing the Jordan canonical form without considering monic polynomials: +3 points.
(iv) Mentioning only the general way of computing the Jordan canonical form and not the specific form in the example: +2 points.

Problem 2

- (a) (i) Correct solution: 5 points.
(ii) Correctly showing that R is nonempty: +1 point.
(iii) Correctly showing that R is closed under addition: +1 points.
(iv) Correctly showing that R is closed: +2 points.

- (v) Showing that R contains the multiplicative identity of $M_{3 \times 3}(K)$: +1 point.
- (b)
 - (i) Correct solution: 10 points.
 - (ii) Correctly showing that I is nonempty: +1 point.
 - (iii) Correctly showing that I is closed under addition: +2 points.
 - (iv) Correctly showing that I is closed under multiplication with elements of R from both sides: +2 points.
 - (v) Identifying correctly a ring S isomorphic to R/I : +2 points.
 - (vi) Finding a ring homomorphism $\phi : R \rightarrow S$ or $\phi : R/I \rightarrow S$: +1 point.
 - (vii) Computing correctly the kernel of ϕ : +1 point.
 - (viii) Computing correctly the image of ϕ : +1 point.
- (c)
 - (i) Correct solution: 10 points.
 - (ii) Identifying correctly a nonzero nilpotent two-sided ideal: +2 points.
 - (iii) Identifying correctly a maximal two-sided ideal: +2 points.
 - (iv) Motivating correctly why the ring is not semisimple: +2 points.
 - (v) Motivating correctly why the ring is noetherian: +2 points.
 - (vi) Motivating correctly why the ring is artinian: +2 points.

Problem 3

- (a)
 - (i) Correct solution: 13 points.
 - (ii) Correctly mentioning the definition of free modules: +1 points.
 - (iii) Having an incomplete argument that $K[X]$ is a free $K[X]$ -module: +3 points.
 - (iv) Correctly showing that $K[X]$ is free as a $K[X]$ -module: +7 points.
 - (v) Correctly mentioning the definition of simple modules: +1 points.
 - (vi) Having an incomplete argument that $K[X]$ is not a simple $K[X]$ -module: +3 points.
 - (vii) Correctly showing that $K[X]$ is not simple as a $K[X]$ -module: +6 points.
- (b)
 - (i) Correct solution: 12 points.
 - (ii) Showing that N is free as a K -module: +2 points.
 - (iii) Having an incomplete argument that N is not free as a $K[X]$ -module: +3 points.
 - (iv) Correctly showing that N is not free as a $K[X]$ -module: +6 points.
 - (v) Showing that N is simple as a K -module: +2 points.
 - (vi) Having an incomplete argument that N is simple as a $K[X]$ -module: +3 points.
 - (vii) Correctly showing that N is simple as a $K[X]$ -module: +6 points.

Problem 4

- (a)
 - (i) Correct solution: 5 points.
 - (ii) Describing the decomposition of $M \cong R^s \oplus \frac{R}{(a_1)} \oplus \cdots \oplus \frac{R}{(a_u)}$ for some $s \geq 0$ and $a_1, \dots, a_u \in R$: +3 points.
 - (iii) Mentioning that the a_i can be taken to be non-units such that $a_1 \mid \cdots \mid a_u$: +1 point.
 - (iv) Mentioning uniqueness up to reordering and multiplication by units: +1 point.
- (b)
 - (i) Correct solution: 10 points.

- (ii) Mentioning or showing that $\text{Tor}M \cong \frac{R}{(a_1)} \oplus \cdots \oplus \frac{R}{(a_u)}$: +3 points.
 - (iii) Showing that $\frac{M}{\text{Tor}M} \cong R^s$ for some s : +5 points.
 - (iii) Mentioning or showing that R^s is free: +2 points.
- (c)
- (i) Correct solution: 10 points.
 - (ii) Showing that \mathbb{Q} is not finitely-generated as a \mathbb{Z} -module: +5 points.
 - (iii) Showing that $\frac{\mathbb{Q}}{\text{Tor}M}$ is not free as a \mathbb{Z} -module: +5 points.
 - (iv) Showing that $\frac{\mathbb{Q}}{\text{Tor}\mathbb{Q}} \cong \mathbb{Q}$: +2 points.
 - (v) Showing that a \mathbb{Z} -basis of \mathbb{Q} must have at most one element: +2 points.
 - (vi) Showing that \mathbb{Q} can not be generated as a \mathbb{Z} -module by one element: +1 points.