Norwegian University of
Science and Technology

Department of Mathematical Sciences

Examination paper for **MA3201 Rings and Modules**

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Examination date: November 28th, 2018

Examination time (from–to): 09:00 – 13:00

Permitted examination support material: D: No printed or hand-written support material is allowed. A simple calculator is allowed.

Other information:

- All answers have to be justified, and they should include enough details in order to see how they have been obtained.
- All sub-problems carry the same weight for grading.
- Assume all rings have unity (all rings have a 1).
- $\mathbb{Z}$ denotes the ring of integers; $\mathbb{R}$ denotes the field of real numbers.

Language: English

Number of pages: 1

Number of pages enclosed: 0

Checked by:

Informasjon om trykking av eksamensoppgave
Originalen er:
1-sidig ☐ **2-sidig** ☒
sort/hvit ☒ **farger** ☐
skal ha flervalgskjema ☐

Date Signature

Notes: All rings have unity (all rings have a 1). $\mathbb{Z}$ denotes the ring of integers; $\mathbb{R}$ denotes the field of real numbers. All answers must be justified. Each sub-problem is worth the same for grading.

Problem 1. Let R be a ring.

(a) Let M and N be R-modules and let $f : M \to N$ be an R-module homomorphism. Prove that $\ker(f)$ is an R-submodule of M.

(b) Let M and N be simple R-modules. If $f : M \to N$ is a nonzero R-module homomorphism, prove that f is an isomorphism.

(c) Let R be a nonzero commutative ring. If M is a maximal ideal of R, prove that M is a prime ideal.

Problem 2.

(a) Prove that $\mathbb{Z}/6\mathbb{Z}$ is isomorphic (as a ring) to $\mathbb{Z}/2\mathbb{Z} \times \mathbb{Z}/3\mathbb{Z}$.

(b) Show that $\mathbb{Z}/6\mathbb{Z}$ is a free $\mathbb{Z}/6\mathbb{Z}$-module but is not a free $\mathbb{Z}$-module.

Problem 3.

(a) Prove that $\mathbb{Z}$ is noetherian but is not artinian.

(b) Prove that $\mathbb{R}[x]$ is noetherian.

(c) Consider the rings $R_1 = \begin{bmatrix} \mathbb{R} & \mathbb{R} \\ 0 & \mathbb{R} \end{bmatrix}$ and $R_2 = \begin{bmatrix} \mathbb{R} & 0 \\ 0 & \mathbb{R} \end{bmatrix}$. Determine whether each of these rings is semisimple (and prove your answer).

Problem 4.

(a) Let $A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 2 & 4 \\ 1 & 4 & 1 \end{bmatrix}$. Find the Smith normal form of A over $\mathbb{Z}$.

(b) Let B be a 6×6 matrix over $\mathbb{R}[x]$ with invariant factors $f_1(x) = x + 1$, $f_2(x) = (x + 1)(x + 3)$, and $f_3(x) = (x + 1)(x + 3)^2$. Give the rational canonical form of B.