4. Ring homomorphisms (Chapter 10.2)

R, S – rings

<u>Definition 4.1</u> (1) A map $f: R \to S$ is called a <u>ring homomorphism</u> if for all $r, s \in R$ we have that

(i) $f(r+s) = f(r) + f(s)$, and
(ii) $f(rs) = f(r) f(s)$

both hold.

(2) For a ring homomorphism $f: R \to S$ the following are equivalent (exercise)

(i) f is bijective.
(ii) There exists a ring homomorphism $g: S \to R$ such that $g \circ f = id_R$, $f \circ g = id_S$.

In this case we say that f is an <u>isomorphism</u> and we write $R \cong S$.

From the point of view of ring theory, isomorphic rings are the same.