<u>Definition 12.6.</u> Assume that R is commutative and M is finitely-generated and free. The number of elements in a basis of M is called the <u>rank</u> of M, denoted $\operatorname{rank}(M)$.

<u>Example 12.7.</u> Let k be a field and V a finite-dimensional k-vector space. Then $\operatorname{rank}(V) = \dim_k(V)$.

<u>Lemma 12.8.</u> Let M be a finitely-generated R-module. Then there exists $n > 0$ and a submodule $K \subseteq R^n$ such that $M \cong R^n/K$.

<u>Proof.</u> Let $M = (m_1, \dots, m_n)$. Define

$$\begin{array}{rccc} \varphi : & R^n & \longrightarrow & M \\ & \rotatebox{90}{$\in$} & & \rotatebox{90}{$\in$} \\ & (r_1, \dots, r_n) & \longmapsto & r_1 m_1 + \dots + r_n m_n . \end{array}$$

It is straightforward to check that φ is a homomorphism of left R-modules. Since $\{m_1, \dots, m_n\}$ generates M, we have that φ is surjective. Set $K = \ker \varphi$. Then by Theorem 10.5 we obtain $R^n/K \cong M$. $\square$