10 Module homomorphisms (Chapter 14.3)

R - ring
M, N - (left) R-modules

<u>Definition 10.1</u> (1) A map $f: M \to N$ is called an <u>R-module homomorphism</u> or <u>R-homomorphism</u> if for all $m_1, m_2, m \in M$ and for all $r \in R$ we have that

(i) $f(m_1 + m_2) = f(m_1) + f(m_2)$, and
(ii) $f(rm) = rf(m)$

both hold. If $N = M$, then f is also called an <u>endomorphism</u> of M.
(2) For an R-homomorphism $f: M \to N$ the following are equivalent (exercise)
(i) f is bijective.
(ii) There exists an R-homomorphism $g: N \to M$ such that $g \circ f = id_M$ and $f \circ g = id_M$.
In this case we say that f is an <u>R-isomorphism</u> and write $M \cong N$.

Isomorphic R-modules have essentially the same structure.