

PROBLEMS ABOUT PRINCIPAL IDEAL DOMAINS

Let R be a PID.

- (1) Let T be a torsion R -module and let p be a prime of R . Prove that if $px = 0$ for some nonzero $x \in T$, then $\text{Ann}(T) \subseteq (p)$.
- (2) Give an example of a PID R and a nonzero torsion R -module M such that $\text{Ann}(M) = 0$.
- (3) If M is a finitely generated torsion R -module, prove $\text{Ann}(M) \neq 0$.
- (4) If M is a finitely generated R -module, prove that $M/\text{Tors}(M)$ is a free R -module.
- (5) Consider the \mathbb{Z} -module $M = \mathbb{Z}^3 \oplus (\mathbb{Z}/10\mathbb{Z}) \oplus (\mathbb{Z}/24\mathbb{Z})$. What are the invariant factors of M ? What are the elementary divisors of M ? What is $\text{Tors}(M)$ isomorphic to? What is the free rank of M ?
... (*more to come*)