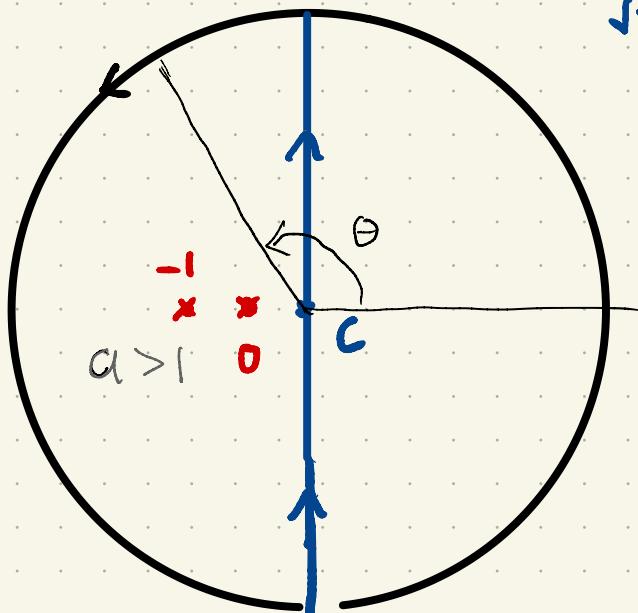


$$\text{INTEGRAL} \quad \frac{1}{2\pi i} \int_{c-i\infty}^{c+i\infty} \frac{a^z}{z(z+1)} dz = \begin{cases} 1 - \frac{1}{a}, & a \geq 1 \\ 0, & 0 < a \leq 1 \end{cases} \quad (c > 0)$$



Radius = $R \gg 1$.

$$a^z = e^{z \log(a)}$$

$$z = c + Re^{i\theta}$$

$$|a^z| = a^c e^{R \cos(\theta) \cdot \log(a)}$$

$$\left| \cos \theta \right| < 0 \quad \text{when } \frac{\pi}{2} < \theta < \frac{3\pi}{2}$$

$$\frac{1}{2\pi i} \int_{\text{Left half-circle}} \frac{a^z}{z(z+1)} dz = \text{Res}_{z=0} \left\{ \frac{a^z}{z(z+1)} \right\} + \text{Res}_{z=-1} \left\{ \frac{a^z}{z(z+1)} \right\}$$

$$= \frac{a^0}{0+1} + \frac{a^{-1}}{(-1)} = 1 - \frac{1}{a} \quad R \gg c+1$$

Left half-circle,

$$a > 1.$$

Integral on the half-circle

$$\left| \frac{1}{2\pi i} \int_{\text{half-circle}} \dots dz \right| \leq \frac{a^c}{2\pi} \int_{\pi/2}^{3\pi/2} \frac{e^{R \cos \theta \log(a)}}{(R-c)(R-c-1)} R d\theta$$

$\rightarrow 0$ as $R \rightarrow \infty$.

The case $a \geq 1$ is done.

The case $a < 1$ is similar (use right half-circle).