Exercise. On the interval $[0,1]$ consider the differential operator

$$Lu = -\Big((1+x^2)u'\Big)',$$

where $u, u' \in AC[0,1]$. Let A_1 be the maximal operator with

$$\mathcal{D}(A_1) = \{u \in AC[0,1] \mid u' \in AC[0,1], A_1 u = Lu \in L^2[0,1]\}.$$

Let $\tilde{A}$ be the self-adjoint Sturm–Liouville operator with

$$\mathcal{D}(\tilde{A}) = \{u \in \mathcal{D}(A_1) \mid u(0) = u'(0), u(1) = -u'(1)\}.$$

Find the Green function $G(x,y)$ associated to $\tilde{A}$.