**Exercise.** Let $A$ and $B$ be two densely defined linear operators in the Hilbert space $H$, with domains of definition $\mathcal{D}(A)$ and $\mathcal{D}(B)$, respectively. We write $A \prec B$ if $\mathcal{D}(A) \subseteq \mathcal{D}(B)$ and $Ax = Bx$ for $x \in \mathcal{D}(A)$. Prove the following statements:

$(i)$ $A$ symmetric $\Leftrightarrow A \prec A^*$.

$(ii)$ $A \prec B \Rightarrow B^* \prec A^*$.

$(iii)$ If $A$ is symmetric, and $B$ is a symmetric extension of $A$, then $B \prec A^*$.

$(iv)$ If $A$ is symmetric and $\mathcal{D}(A) = H$, then $A$ is self-adjoint.