NTNU
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Department of Mathematical Sciences

Examination paper for **MA3002 General Topology**

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Problem 1

a) Let X and Y be topological spaces, and $A \subseteq X$ and $B \subseteq Y$.

Show that $\overline{A \times B} = \bar{A} \times \bar{B}$ in $X \times Y$.

b) Show that X is a Hausdorff space if and only if the diagonal $\Delta = \{(x, x) \mid x \in X\}$ is closed in $X \times X$.

Problem 2 Let $\mathcal{T}$ be the topology on $\mathbb{R}$ (the real numbers) given by the basis $\{[a, b) \mid a < b \text{ og } a, b \in \mathbb{Q}\}$. $\mathbb{Q}$ denotes here the rational numbers. Find the closure of the following subspaces of $(\mathbb{R}, \mathcal{T})$.

a) $(0, \sqrt{2})$

b) $(\sqrt{2}, 3)$

Problem 3

a) Let X be a topological space and $A \subseteq X$. Let $\{x_n\}$ be a sequence in A.

Show that if $x_n \to x$ then $x \in \bar{A}$.

Show that the opposite also holds if X is metrizable.

b) Let $f \colon X \to Y$ be a continuous map between two topological spaces.

Show that $x_n \to x$ implies $f(x_n) \to f(x)$. Furthermore, show that the opposite also holds if X is metrizable.

Problem 4

a) Give a definition of a connected topological space.

b) Let A be a connected subspace of X.

Show that if $A \subseteq B \subseteq \bar{A}$ then B is also connected.

Problem 5 A subspace A of a topological space X is dense if $\bar{A} = X$.

a) Let U and V be to open dense subspaces of a space X.

Show that $U \cap V$ is dense. Give an example.

b) Let (X, d) be a metric space and assume that X has a countable dense subspace.

Show that X is 2nd countable (meaning that X has a countable basis for the metric topology).

Problem 6 Let X be a topological space. Assume that for any pair of points $x, y \in X$, $x \neq y$ there exists a continuous map $f\colon X \to [0, 1]$ such that $f(x) = 1$ and $f(y) = 0$.

a) Show that X is a Hausdorff space.

b) Let K be a non-empty compact subspace X, $K \neq X$. Let $x \in X - K$.

Show that there exists a continuous map $g\colon X \to [0, 1]$ such that $g(x) = 1$ and $g(K) \subseteq [0, 1/2)$.