NTNU
Institutt for matematiske fag

MA2501 Numerical Methods

Project 3

Spring 2024

Deadline: Monday, April 15.
You may write solutions in Norwegian or English, as preferable. The most important part is *how* you arrive at an answer, not the answer itself.

Consider the following system of ODEs:

$$x' = -y - z \tag{1}$$
$$y' = x + 0.2y \tag{2}$$
$$z' = 0.2 + z(x - 5.7) \tag{3}$$

1

a) Show that the system is locally Lipschitz and find a local Lipschitz constant for $-100 \leq x, y, z \leq 100$.

b) Implement and use improved Euler with $h = 0.01$ to solve the system with the initial conditions $x_0 = y_0 = 10, z_0 = 0$ up to $t = 1000$. Plot the three solution functions as functions of time.

c) Do the task above with $x_0 = 10.000001$ instead. Plot the difference between the two solutions as a function of time, also up to $t = 1000$.

In a game of shuffleboard (a game similar to curling) it is beneficial to be able to propel a stone towards a target such that it stops on top of the target.

We consider a variant played with a light polystyrene stone where air resistance matters. The organizers have also made the game more difficult by gradually decreasing friction along the court in an exponential fashion.

We can model the motion of the stone with the following differential equation where d is a drag coefficient and r is a friction coefficient.

$$y'' = -d|y'|y' - re^{-y}y' \tag{4}$$

We use $d = r = 3$ for this problem.

2

a) Rewrite the second order ODE as a system of first order ODEs.

b) Suppose you impart some initial velocity y_0' on the object from the initial position $y_0 = 0$. Implement improved Euler and use it with $h = 0.0001$ to solve the differential equation on the interval $t \in [0, 10]$ for $y_0' = 0$ and $y_0' = 1000$. Plot both solutions.

c) Suppose our target is located at 2, use bisection to find a starting velocity y_0' that allows us to hit the target at $t = 10$, and plot the trajectory of the stone for this starting velocity.

d) Explain how the technique outlined above can be used to solve nonlinear boundary value problems. (Which boundary value problem did we solve, and how, and how does it generalize to other boundary value problems?)