NTNU
Institutt for matematiske fag

MA2501 Numerical Methods

Exercise set 4

Spring 2024

Deadline: Monday 22. April
You may write solutions in Norwegian or English, as preferable. The most important part is *how* you arrive at an answer, not the answer itself.

1

a) Consider the following ODE on $\mathbb{R}$:

$$-\frac{d}{dx}\left((x^2+1)\frac{du}{dx}\right)+(4x^4-2x^2)u=2e^{-x^2} \quad , \quad x\in\mathbb{R} \tag{1}$$

Show by direct insertion that $u(x)=e^{-x^2}$ is the solution.

b) Consider the same ODE, but this time with boundary conditions $u(\pm 2)=e^{-4}$ Use a self-adjoint finite difference scheme for approximating the solution using step size $h=4/N$, where $N\in\{2^m\}_{m=3}^{10}$. Plot the relative discrete L^2-error in a loglog-plot, with N on the x-axis and error on the y-axis.

2

a) Consider Chebyshev's equation on $[-1,1]$:

$$(1-x^2)T_n''-xT_n'+n^2T_n=0 \quad , \quad x\in[-1,1] \tag{2}$$

Rewrite it as a Sturm-Liouville eigenvalue problem.

b) Use a self-adjoint finite difference scheme for approximating the solution using step size $h=2/N$, where $n\in[3,6]$ and $N\in\{2^m\}_{m=3}^{12}$. Plot the relative discrete L^2-error in a loglog-plot, with N on the x-axis and error on the y-axis. Utilize the relations $T_n(\pm 1)=(\pm 1)^n$ and $T_n(x)=\cos(n\cos^{-1}x)$.

Hint: If the code for task 1b) works, you can reuse it for task 2b) with some modifications and extensions.