NTNU
Institutt for matematiske fag

MA2501 Numerical Methods

Exercise set 2

Spring 2024

Deadline: Monday 26. February
You may write solutions in Norwegian or English, as preferable. The most important part is *how* you arrive at an answer, not the answer itself.

1 Consider $D_m = \{(x_i, y_i) \,:\, 0 \leq i \leq m\}$ and $p_n(x) = a_0 + a_1 x + \cdots + a_n x^n$.

a) Derive the linear system of equations $\mathbf{Ma} = \mathbf{b}$ that occurs when we use p_n to approximate D_m through least-squares.

b) Describe an algorithm for assembling $\mathbf{M}$ and $\mathbf{b}$ with minimal running time.
Hint: Exploit the symmetry of $\mathbf{M}$ and reduce the number of multiplications on both sides of the equation system.

c) Consider the data sets `Data1.txt` and `Data2.txt`. Approximate the first one with p_1, p_3, p_5, and the second with p_2, p_4, p_6. Compute the error in the l_2-norm and determine which approximation is best for these data sets. Compare the discrete data and the polynomial approximation in one figure for each data set.
Hint: Use the variable `line` as a for-loop variable when iterating the content of the txt-files and separate the data with `line.split(" ")`

2 Consider the following data set:

x_i	0	1	3	5	6
$f(x_i)$	36	8	-5	11	28

a) Use divided differences to derive the Newton interpolation polynomial for these data. Compute this polynomial at $x = 2$.

b) Repeat the same procedure above without $(3, -5)$.

3 Consider the function $f(x) = x^4 e^{-x^2}$, where $x \in [-4, 4]$.

a) Write a Python program that plots the corresponding interpolation polynomial $p(x)$ using n shifted Chebyshev points, $n = \{2i+1\}_{i=1}^{5}$. Collect all of them in a single plot and compare with the exact solution.
Hint: Find the Newton interpolation coefficients once, store them in an array, and use it when evaluating the polynomials on Newton form.

b) Write a program that computes the discrete L^2-error between f and p on $[-4, 4]$ using shifted Chebyshev points, $n = \{2i+1\}_{i=1}^{10}$. Display them in a graph.
Hint: Choose 401 equidistant points on $[-4, 4]$.

Throughout this assignment, you will need the discrete l^2-norm for computing error between function approximations:

$$\|f - p\|_{l^2} = m^{-1} \sum_{i=1}^{m} |f(x_i) - p(x_i)|^2$$