MA2501
NUMERICAL METHODS
NTNU, SPRING 2020

Exercise Set 1

Exercise sets are to be handed in individually by each student using the OVSYS system[1]. Each set will be graded (godkjent / ikke godkjent). The three exercise sets are mandatory to be admitted to the final exam. Note that to get an exercise set approved you are required to have solved correctly at least 70% (providing detailed arguments/computations) and made serious and substantial attempts at solving at least 90% of a given exercise set.

You are asked to pay attention to the quality of presentation, in particular, the correctness of mathematical notation and formalism.

Deadline: 31-01-2022, 16h00 (OVSYS)

Problem 1. (1) *Let $x = 0.d_1 \cdots d_k \cdots \times 10^n$ in decimal representation (basis $b = 10$). Aiming at a k-digit floating point representation, we consider chopping instead of rounding, i.e., we keep the k first digits and throw away the rest*

$$f\ell(x) = 0.d_1 \cdots d_k \times 10^n$$

Show that 10^{-k-1} is a bound for the relative error when using k-digit chopping.

(2) *Let s be a parameter. Show that the function $f(t) = t^3 + 2t + s$ crosses the t-axis exactly once for any value of s.*

(3) *Recall that Taylor's polynomial $p(t)$ is determined by requiring that the values of the polynomial and its first n derivatives match those of a given function $f(t)$ at a single argument t_0, i.e., $p^{(i)}(t_0) = f^{(i)}(t_0)$, for $0 \le i \le n$. Find a formula for $R(t, t_0) = f(t) - p(t)$ in integral form. Assume that $f^{(n+1)}(t)$ is continuous between t and t_0.*

(4) *Determine the Taylor polynomial $P_n(t)$ for $n = 2$ for the function $f(t) = \exp(t)\cos(t)$ around the point $t_0 = 0$. Find an upper bound for the remainder term for $t = 0.5$.*

Problem 2. *Consider the equation $t^2 = a$ written in fixed point form $t = F(t)$. It turns out that several $F(t)$ are possible:*

$$F_1(t) = 0.5(t + at^{-1}) \quad F_2(t) = at^{-1} \quad F_3(t) = 2t - at^{-1}.$$

Verify that this is true and discuss the (non-)convergence behaviour for the corresponding iteration $t_{n+1} = F(t_n)$, $n \ge 0$, for each of the three cases. If possible, determine the order of convergence.

Problem 3. Süli–Mayers*: Ex. 1.8, Ex. 2.8, Ex. 4.8.*

Problem 4. (Optional) *Solve problems in the Jupyter notebook "PythonExercises-Updated.ipynb". You do not need to submit this part, but we recommend you to do this especially if you are not experienced with Python coding.*

Date: January 17, 2022.

[1]A link can be found here: `https://wiki.math.ntnu.no/ma2501/2022v/start`