MA2501, Spring 2020, Numerical Methods

February 2020

Problem 1

Apply Lagrange interpolation to solve the following problem: find the unique polynomial of degree at most two satisfying

$$p_2(0) = 0,\ p_2\left(\tfrac{2}{3}\right) = 1,\ p_2(1) = 0.$$

Guidelines: We use the Lagrange form for the interpolation polynomial:

$$p_2(x) = \sum_{i=0}^{2} y_i\, \ell_i(x), \quad \ell_i(x) := \prod_{j=0, j\neq i}^{2} \frac{x - x_j}{x_i - x_j},$$

and with $x_0 = 0$, $x_1 = \frac{2}{3}$, $x_2 = 1$, $y_0 = y_2 = 0$ $y_1 = 1$.

Problem 2

Construct the Lagrange interpolation polynomial p_1 of degree 1 for a continuous function f defined on the interval $[-1, 1]$, using the interpolation points $x_0 = -1$ and $x_1 = 1$. Show further that if the second order derivative of f exists and is continuous on $[-1, 1]$ then

$$|f(x) - p_1(x)| \leq \frac{M_2}{2}(1 - x^2) \leq \frac{M_2}{2}, x \in [-1, 1],$$

where

$$M_2 = \max_{x\in[-1,1]} |f''(x)|.$$

Give an example of a function f for which equality is achieved.

Guidelines:

- Use the definition of Lagrange interpolation polynomial to find p_1.
- Use the error bound for the interpolation polynomial (Theorem 6.2).
- To find a concrete example for f, try with a quadratic polynomial of the form $f(x) = ax^2 + bx + c$ and determine a, b and c.

Problem 3

Write down the Lagrange interpolation polynomial of degree 1 for the function $f : x \mapsto x^3$, using the points $x_0 = 0$ and $x_1 = a$. Verify Theorem 6.2 by direct calculation showing that in this case ξ is unique and has the value $\xi = \frac{1}{3}(x + a)$.