This exam set includes an appendix with formulae and theorems useful for the solution of the exam questions.

1 APPENDIX

This appendix contains useful formulae and theorems to solve the exam questions.

Intermediate value theorem

Theorem *Suppose that f is a real-valued function, defined and continuous on the closed interval $[a, b]$ of $\mathbb{R}$. Then, f is a bounded function on the interval $[a, b]$ and, if y is any number such that*

$$\inf_{x\in[a,b]} f(x) \leq y \leq \sup_{x\in[a,b]} f(x)$$

then there is a number $\xi \in [a, b]$ such that $f(\xi) = y$. In particular the infimum and supremum of f are achieved, and can be replaced by $\min_{x\in[a,b]}$ and $\max_{x\in[a,b]}$, respectively.

Taylor's Theorem

Theorem *Suppose that n is a nonnegative integer, and f is a real-valued function, defined and continuous on the closed interval $[a, b]$ of $\mathbb{R}$, such that the derivatives of f of order up to and including n are defined and continuous on the closed interval $[a, b]$. Suppose further that $f^{(n)}$ is differentiable on the open interval (a, b). Then, for each value of x in $[a, b]$, there exist a number $\xi = \xi(x)$ in the open interval (a, b) such that*

$$\begin{aligned} f(x) = f(a) + (x-a)f'(a) + \cdots + \frac{(x-a)^n}{n!} f^{(n)}(a) + \\ + \frac{(x-a)^{n+1}}{(n+1)!} f^{(n+1)}(\xi). \end{aligned}$$

Error formula for the interpolation polynomial

Suppose x_i, $i = 0, \dots, n$ are distinct real numbers. Suppose $p_n(x)$ is the interpolation polynomial of degere less than or equal to n interpolating the data $(x_0, f(x_0)), \dots, (x_n, f(x_n))$ on the interpolation nodes $x_0, \dots, x_n$.

Theorem *Suppose that* $n \geq 0$*, and that* f *is a real valued function, defined and continuous on the closed real interval* $[a, b]$*, such that the derivative of* f *and of order* $n+1$ *exists and is continuous on* $[a, b]$*. Then, given that* $x \in [a, b]$*, there exists* $\xi = \xi(x)$ *in* (a, b) *such that*

$$f(x) - p_n(x) = \frac{f^{(n+1)}(\xi)}{(n+1)!}\,\pi_{n+1}(x),$$

where

$$\pi_{n+1}(x) = (x - x_0) \cdot (x - x_1) \cdots (x - x_n).$$