



Norwegian University of Science
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Department of Mathematics

MA2501: Numerical Methods
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Exercise set 2

- 1 Prove that a subordinate matrix norm is a matrix norm, i.e. if $\|\cdot\|$ is a vector norm on \mathbb{R}^n , then $\|\mathbf{A}\| = \max_{\|\mathbf{x}\|=1} \|\mathbf{A}\mathbf{x}\|$ is a matrix norm.

- 2 Solve the linear systems in exercise set 1, task 4, using Gaussian elimination with scaled partial pivoting. Write down all row interchanges.

- 3 Cf. Cheney and Kincaid, Exercise 2.2.24
Solve the following systems using Gaussian elimination without pivoting, with partial pivoting, with scaled partial pivoting, and with complete pivoting, carrying only four significant digits. Also, find the true solution:

a)	b)
$0.004000x + 69.13y = 69.17,$	$30.00x + 591400y = 591700,$
$4.281x - 5.230y = 41.91.$	$5.291x - 6.130y = 46.78.$

- 4 Implement Gaussian elimination without pivoting and with scaled partial pivoting, in PYTHON. Test your functions on the linear systems of exercise set 1, task 4.

5 Cf. Cheney and Kincaid, Computer Exercise 2.2.14.

The determinant of a matrix can be easily computed with an algorithm for the forward elimination part of Gaussian elimination. This is due to the facts that:

- The determinant of a triangular matrix is the product of its diagonal entries.
- The determinant of a matrix does not change when the multiple of a one row is added to another row.
- If two rows of a matrix are exchanged, then the determinant changes sign.

Now note that forward elimination can be interpreted as a method for reducing a matrix to (upper) triangular form by only exchanging rows and adding multiples of one row to another row.

Write a function in PYTHON `Compute_Determinant(A)` for the computation of the determinant of an $n \times n$ matrix \mathbf{A} , and test the function on the following test matrices with several values of n .

a)

$$a_{ij} = |i - j| \quad \text{Det}(\mathbf{A}) = (-1)^{n-1}(n-1)2^{n-2}$$

b)

$$a_{ij} = \begin{cases} 1 & j \geq i \\ -j & j < i \end{cases} \quad \text{Det}(\mathbf{A}) = n!$$

c)

$$\begin{cases} a_{1j} = a_{j1} = n^{-1} & j \geq 1 \\ a_{ij} = a_{i-1,j} + a_{i,j-1} & i, j \geq 2 \end{cases} \quad \text{Det}(\mathbf{A}) = n^{-n}.$$