Exercise set 0

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This set of exercises is meant to give a short introduction into the usage of PYTHON.

1 Linear algebra and plotting:

Find and plot the 3rd order polynomial interpolating the following given points:

i	1	2	3	4
x_i	-2	0	1	3
y_i	-16	-3	-1	24

In other words: Find a polynomial

$$p(x) = a_3x^3 + a_2x^2 + a_1x + a_0$$

that satisfies $p(x_i) = y_i$ for $i = 1,\ 2,\ 3,\ 4$.

a) Verify that the coefficients satisfy the linear system

$$\begin{pmatrix} 1 & -2 & 4 & -8 \\ 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 \\ 1 & 3 & 9 & 27 \end{pmatrix} \begin{pmatrix} a_0 \\ a_1 \\ a_2 \\ a_3 \end{pmatrix} = \begin{pmatrix} -16 \\ -3 \\ -1 \\ 24 \end{pmatrix}.$$

b) Use PYTHON to solve the linear system.

c) Use PYTHON for plotting the interpolation polynomial.

PYTHON resources

PYTHON is already installed at certain computer labs at NTNU. It is also highly recommended that you install it on your own computer, in particular the software Anaconda, which contains all the packages required for scientific computations in PYTHON. Instructions for that can be found at `http://continuum.io/downloads`.

The documentation of PYTHON is very extensive. We recommend the websites and books posted on the main page of the course.

Plotting

Plotting requires the library *matplotlib.pyplot*, and the appearance of the plots can be improved by using several variables and commands like:

`color`	color of the graph plotted
`linestyle`	line style width of the graph plotted
`grid()`	plot a grid over the function plot
`set_xlabel('x')`	annotation of the x-axis
`set_ylabel('y')`	annotation of the y-axis
`legend()`	legend for all the graphs

2 Some simple programming:

Euler's number e can, for instance, be computed using either of the formulas

$$e = \lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n \qquad \text{or} \qquad e = \sum_{k=0}^{\infty}\frac{1}{k!}$$

a) Write two PYTHON-programs that compute the numbers

$$a_n = \left(1+\frac{1}{n}\right)^n \qquad \text{or} \qquad b_m = \sum_{k=0}^{m}\frac{1}{k!}$$

for different values of n and m. Compare the results with the true value of e.

b) One of the two methods does not seem to converge to e. Which one? Why?

Programming in PYTHON

PYTHON-files have the ending `.py` and may either be scripts (which simply contain several lines of PYTHON-code that are executed when the file is called) or modules (containing functions we are going to include in the main file).

A simple PYTHON-function for computing the square of a number (or the pointwise square of an array) may consist of the following lines:

```
def function f(x):
return x**2
```

This code has to be stored in a file, like `test.py` (the does not need to have the same name as the function). Then, calling the script with for instance `f(2)`in the terminal or graphical environment gives the result `4`.