Exercise set 9

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1 Compute numerical approximations of the definite integral

$$\int_0^2 e^{-x^2} \sin(x)\, dx$$

a) using the composite trapezoid rule with $h = 1$ and $h = 1/2$,

b) using the composite Simpson rule with $h = 1$ and $h = 1/2$,

(You may want to use MATLAB for the computations.)

2 Compute numerical approximations of the definite integrals

$$\int_0^1 x^{7/2}\, dx \qquad \text{and} \qquad \int_0^1 x^{5/2}\, dx$$

using the trapezoid method and the Simpson method with $h = 1$, $h = 1/2$, $h = 1/4$.

a) Compute for all approximations the respective approximation errors and try to estimate numerically the convergence rate.

b) Which convergence rate would you usually expect for the trapezoid method and the Simpson method, and which have you actually observed? Try to explain possible discrepancies.

(You may want to use MATLAB for the computations.)

3 Implement and use Romberg integration to approximate the integral in Exercise 1. Compute the Romberg table until $|R(n-1, n-1) - R(n, n)| < 10^{-6}$. Compare your result to the exact value of the integral.

4 Romberg integration is used to approximate $\int_2^3 f(x)dx$. If $f(2) = 0.51342$, $f(3) = 0.36788$, $R(2, 0) = 0.43687$, and $R(2, 2) = 0.43662$, find $f(2.5)$.