Norwegian University of Science and Technology
Department of Mathematics

MA2501 Numerical Methods
Spring 2016

Exercise set 8

1 Consider the function $f(x) = (x^2+1)e^x$.

a) Use central differences, i.e., the formula

$$f'(x) \approx \frac{f(x+h)-f(x-h)}{2h},$$

with step sizes $h=1$, $h=1/2$, $h=1/4$, $h=1/8$, in order to approximate $f'(0)$.

b) Use Richardson extrapolation for obtaining a better approximation of $f'(0)$ from the values you have already computed.

2 Derive an $\mathcal{O}(h^4)$ five point formula to approximate $f'(x_0)$ that uses $f(x_0-h)$, $f(x_0)$, $f(x_0+h)$, $f(x_0+2h)$ and $f(x_0+3h)$. Test the formula on $f(x) = \sin x$ at $x=1$ and try to verify numerically that is has the stated order.

Hint: Consider the expression $Af(x_0-h)+Bf(x_0+h)+Cf(x_0+2h)+Df(x_0+3h)$. Expand in fourth Taylor polynomials, and choose A, B, C and D appropriately.

3 **Cf. Cheney and Kincaid, Exercises 4.1.16–17**

Assume that the function φ has the form

$$\varphi(h) = L - c_1 h - c_2 h^2 - c_3 h^3 - c_4 h^4 - \dots$$

a) Combine the values $\varphi(h)$ and $\varphi(h/2)$ in order to obtain a higher order approximation of L.

b) Try to generalize the idea of Richardson extrapolation to the function φ.

4 Compute numerical approximations of the definite integral

$$\int_0^1 \sin(x^2)\,dx$$

using the composite trapezoid rule with $h=1$, $h=1/2$, and $h=1/4$. In addition, provide estimates for the approximation error.