Norwegian University of Science and Technology
Department of Mathematics

Exercise set 4

1 **a)** Compute the LDL^T and, if possible, Cholesky factorization of the following matrices:

$$A_1 = \begin{pmatrix} -2 & -4 & 6 & -2 \\ -4 & -5 & 12 & -7 \\ 6 & 12 & -17 & 7 \\ -2 & -7 & 7 & 1 \end{pmatrix}, \qquad A_2 = \begin{pmatrix} 1 & 2 & -3 & 1 \\ 2 & 7 & -6 & -1 \\ -3 & -6 & 10 & -2 \\ 1 & -1 & -2 & 6 \end{pmatrix}.$$

b) Use the previously computed factorizations to solve the linear systems $A_j x = b$ with $b = (-3, 3, 1, -3)^T$.

2 How many operations are required for the computation of the Cholesky factorization of a symmetric and positive definite matrix? Compare the result with the numerical complexity of the LU factorization.

3 Write a Matlab program for the solution of a linear system $Ax = b$ with symmetric matrix $A \in \mathbb{R}^{n\times n}$ using the LDL^T factorization. Include some error messages for the situation that A is non-symmetric or the factorization fails.

4 Use the bisection method for finding a solution of the equation $x^2 - 2 = 0$ (i.e., for computing $\sqrt{2}$) starting with the interval $[1, 2]$. Stop the computations when the error is smaller than 10^{-3}. How many iterations would be needed to obtain a result with an error smaller than 10^{-12}?

5 Implement the bisection method in Matlab. Your program should take as an input the two initial points $a < b$, the desired accuracy $\varepsilon > 0$, and a reference to the function f a root of which is to be computed.

Use your program for finding the solution of the equation $\tan(x) = 0$ in the interval $[2, 4]$ (which should be π) and a solution of the equation $3\cos(x)\sin(x^2) = 1$ in the interval $[0, 10]$. What happens if you apply the method to the equation $\tan(x) = 0$ with the starting interval $[1, 2]$?

6 Suppose $f(x)$ is $C[a, b]$ with a sign change on and unique zero in the interior of $[a, b]$. Let $h(x) = f(x)g(x)$ where $g(x)$ is a positive and continous function on $[a, b]$. Let c_f^n and c_h^n be the n-th approximation from the Bisection method to the root of f and h

respectively, with initial interval $[a, b]$. Can we say anything about c_f^n compared to c_h^n?