Norwegian University of Science
and Technology
Department of Mathematics

MA2501 Numerical Methods Spring 2015

Exercise set 2

1 Prove that a subordinate matrix norm is a matrix norm, i.e. if $\|\cdot\|$ is a vector norm on $\mathbb{R}^n$, then $\|\mathbf{A}\| = \max_{\|\mathbf{x}\|=1} \|\mathbf{A}\mathbf{x}\|$ is a matrix norm.

2 Solve the linear systems in exercise set 1, task 5, using Gaussian elimination with scaled partial pivoting. Write down all row interchanges.

3 **Cf. Cheney and Kincaid, Exercise 2.2.24**

Solve the following systems using Gaussian elimination without pivoting, with partial pivoting, with scaled partial pivoting, and with complete pivoting, carrying only four significant digits. Also, find the true solution:

a)

$$\begin{aligned} 0.004000x + 69.13y &= 69.17, \\ 4.281x - 5.230y &= 41.91. \end{aligned}$$

b)

$$\begin{aligned} 30.00x + 591400y &= 591700, \\ 5.291x - 6.130y &= 46.78. \end{aligned}$$

4 Implement in MATLAB Gaussian elimination both without pivoting and with scaled partial pivoting. Test your functions on the linear systems of exercise set 1, task 5.

5 **Cf. Cheney and Kincaid, Computer Exercise 2.2.14.**

The determinant of a matrix can be easily computed with an algorithm for the forward elimination part of Gaussian elimination. This is due to the facts that:

- The determinant of a triangular matrix is the product of its diagonal entries.
- The determinant of a matrix does not change when the multiple of a one row is added to another row.
- If two rows of a matrix are exchanged, then the determinant changes sign.

Now note that forward elimination can be interpreted as a method for reducing a matrix to (upper) triangular form by only exchanging rows and adding multiples of one row to another row.

Write a function in MATLAB `CompDet`($\mathbf{A}$) for the computation of the determinant of an $n \times n$ matrix $\mathbf{A}$, and test the function on the following test matrices with several values of n.

a)

$$a_{ij} = |i-j| \quad \text{Det}(\mathbf{A}) = (-1)^{n-1}(n-1)2^{n-2}$$

.

b)

$$a_{ij} = \begin{cases} 1 & j \geq i \\ -j & j < i \end{cases} \qquad \text{Det}(\mathbf{A}) = n!.$$

c)

$$\begin{cases} a_{1j} = a_{j1} = n^{-1} & j \geq 1 \\ a_{ij} = a_{i-1,j} + a_{i,j-1} & i,j \geq 2 \end{cases} \qquad \text{Det}(\mathbf{A}) = n^{-n}.$$