Norwegian University of Science and Technology
Department of Mathematics

MA2501 Numerical Methods
Spring 2014

Exercise set 10

1 The third order Adams–Bashforth method has the form

$$\begin{aligned} y_{k+1} &= y_k + h\Big(\frac{23}{12}f_k - \frac{16}{12}f_{k-1} + \frac{5}{12}f_{k-2}\Big), \\ t_{k+1} &= t_k + h, \\ f_{k+1} &= f(t_{k+1}, y_{k+1}). \end{aligned}$$

Show that this method can be derived by interpolating the function $\tau \mapsto f(\tau, y(\tau))$ in the points t_{k-2}, t_{k-1}, and t_k and then integrating the resulting quadratic polynomial.

2 Consider the differential equation

$$\begin{aligned} y' &= y - \frac{y^2}{2}, \\ y(0) &= 1. \end{aligned}$$

Compute three steps with step size $h = 1$ using:

a) the second order Adams–Bashforth method,

b) the second order Adams–Moulton method,

c) the second order Adams–Bashforth–Moulton method.

In all three cases, use the improved Euler method for computing the iterate y_1.

3 Assume that the function $f\colon \mathbb{R} \times \mathbb{R} \to \mathbb{R}$ is continuous and decreasing in its second component. That is,

$$f(t, y) \le f(t, z) \qquad \text{whenever} \qquad y \ge z.$$

Show that the implicit Euler method for the solution of the differential equation

$$\begin{aligned} y' &= f(t, y), \\ y(t_0) &= y_0, \end{aligned}$$

is well-defined. That is, regardless of the step-size $h > 0$, the (non-linear) equation that has to be solved in each iteration has a unique solution.

$\boxed{4}$ Compute three steps with step size $h = 1$ for the numerical solution of the differential equation

$$y' = -3y - e^y,$$
$$y(0) = 1,$$

using

a) Euler's method,

b) the improved Euler method,

c) the implicit Euler method. Numerically solve the non-linear equations you obtain by performing two steps of Newton's method in each step of the implicit Euler method.