



[1] Cf. Cheney and Kincaid, Exercise 4.1.9

Consider the data points

x_i	0	1	2	4	6
$f(x_i)$	1	9	23	93	259

- Find the interpolation polynomial through these data points using Newton interpolation, and compute an approximation of f at $x = 3$.
- Do the same using only the first four interpolation points.

[2] Approximate the function $\sin(x)$ on the interval $[0, 1]$ by an interpolation polynomial of degree 4 using:

- equidistant interpolation points,
- Chebyshev interpolation points.

In addition, estimate the quality of the two approximations on the interval $[0, 1]$.

[3] Assume that you want to interpolate the function $f(x) = 2^x$ on the interval $[-2, 2]$ using polynomial interpolation with equidistant interpolation points. How many interpolation points will be needed in order to guarantee that the maximal approximation error on $[-2, 2]$ is smaller than 10^{-5} ? (You might want to use MATLAB at some point.)

[4] Consider the function $f(x) = (x^2 + 1)e^x$.

- Use central differences, i.e., the formula

$$f'(x) \approx \frac{f(x+h) - f(x-h)}{2h},$$

with step sizes $h = 1, h = 1/2, h = 1/4, h = 1/8$, in order to approximate $f'(0)$.

- Use Richardson extrapolation for obtaining a better approximation of $f'(0)$ from the values you have already computed.

5 Cf. Cheney and Kincaid, Exercises 4.1.16–17

Assume that the function φ has the form

$$\varphi(h) = L - c_1 h - c_2 h^2 - c_3 h^3 - c_4 h^4 - \dots$$

- a) Combine the values $\varphi(h)$ and $\varphi(h/2)$ in order to obtain a higher order approximation of L .
- b) Try to generalize the idea of Richardson extrapolation to the function φ .

6 Write a MATLAB-program for the approximation of derivatives using central differences and Richardson extrapolation. Your program should take as an input a function f , a point x where you want to approximate f' , a basic step size h , and the desired approximation order.

Test your program on the function \sin with $x = \pi/3$ and on the function from exercise

4. For which parameters do you obtain the best results?