



Department of Mathematical Sciences

## Examination in **MA1301 Midterm in Number Theory**

**For questions during the exam:** Franz Luef/Magnus Landstad

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**Examination date:** 9 October 2013

**Time (from-to):** 10.15–11:45

**Aid code/Allowed aids:** C

Specified calculator (Citizen SR-270X or HP 30S)

No other aids have been specified

**Language:** English

**Number of pages:** 1

**Number of additional pages:** 0

**Kontrollert av:**

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Note! Students will find their grades in "Studentweb". If you have questions regarding your grades, you must contact the department. The examination office cannot answer such questions.

Give reasons for all answers.

**Problem 1** Use the principle of mathematical induction to show that

$$\binom{2}{2} + \binom{3}{2} + \cdots + \binom{n}{2} = \binom{n+1}{3}.$$

**Problem 2** Find the greatest common divisor of 326 and 78, and find integers  $x$  and  $y$  such that  $\gcd(326, 78) = 326x + 78y$ .

**Problem 3** State the Chinese Remainder Theorem for three congruences and use it to solve the following system of congruences

$$\begin{aligned}x &\equiv 1 \pmod{3} \\x &\equiv 2 \pmod{5} \\x &\equiv 3 \pmod{7}\end{aligned}$$

**Problem 4** Compute  $2^{32} \pmod{37}$  both by repeated squaring and by the help of Fermat's Little Theorem. (For the second method use that  $2^{36} = 2^4 2^{32}$ .)

**Problem 5** Prove that  $\sqrt{5}$  is an irrational number.