



Norwegian University of
Science and Technology

Department of Mathematical Sciences

Examination paper for **MA1202/MA6202 Linear Algebra with Applications**

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Examination date: June 9th, 2021

Examination time (from-to): 09:00 – 13:00

Permitted examination support material: A

All printed and hand-written support material is allowed. All calculators are allowed.

Other information:

You may write in English or Norwegian.

All answers have to be justified.

No questions can be answered simply with a yes, a no, or a number. You must yourself add necessary assumptions if needed. The answer must include enough details in order to see how they have been obtained.

All 10 sub-problems carry the same weight for grading.

Language: English

Number of pages: 2

Number of pages enclosed: 0

Checked by:

Informasjon om trykking av eksamensoppgave

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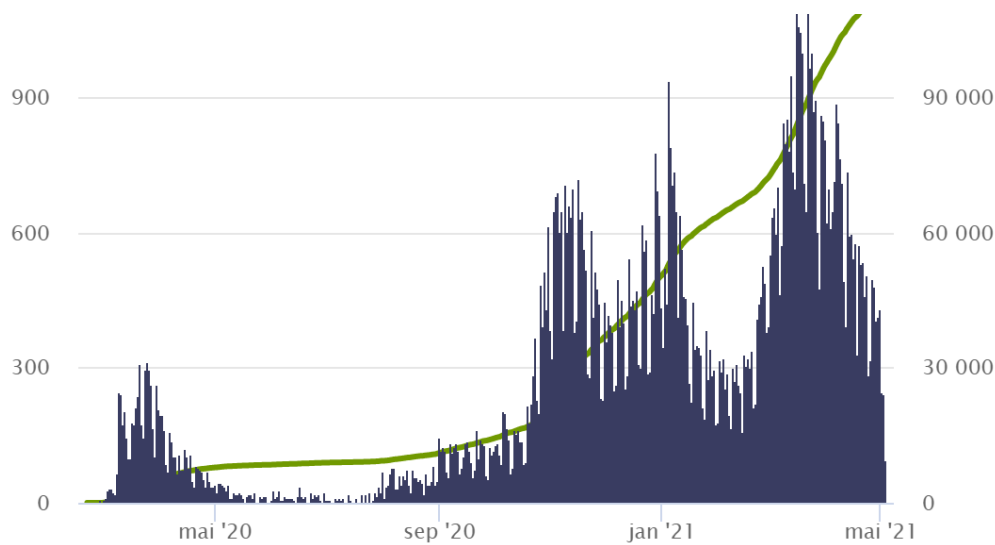


Figure 1: Reported numbers of Covid-19 cases.

Problem 1

Let $x_1 < x_2 < \dots < x_n$ and assume f_1, \dots, f_n to be functions with $f_i(x_i) = 1$ and $f_i(x_j) = 0$ for $i \neq j$.

a) Is $\{f_1, \dots, f_n\}$ linearly independent?

Let f_1, \dots, f_n be polynomials of degree less than n and fulfilling the assumption.

b) Which polynomials can be written on the form $c_1 f_1 + \dots + c_n f_n$?

Figure 1 shows the reported numbers of Covid-19 cases in Norway including Monday May 3th, 2021. Let $g(x)$ be the number of reported cases on date x . On May 3th the number was $g(\text{May 3th, 2021}) = 97$. Assume that you from the reported number of Covid-19 cases have calculated $a_i = f(x_i) = \ln(g(x_i) + 0.1)$ for selected dates x_1, \dots, x_{10} corresponding to observations on Mondays.

c) How can a polynomial p be found so that $a_i = p(x_i)$ for all x_1, \dots, x_{10} ?

d) How can the projection theorem and an adjoint matrix be used to find a third order polynomial q approximating f ?

e) Would you use p or q to predict the daily number of reported Covid-19 cases on the remaining Mondays in May?

In Problem 2

$$Dx(t) = x'(t) = \lim_{\delta \rightarrow 0} \frac{x(t + \delta) - x(t)}{\delta}$$

and V is the set of functions $x : \mathbb{R} \rightarrow \mathbb{C}$ so that $D^n x$ exists for all $n \in \mathbb{N}$. The symbols \mathbb{N} , \mathbb{R} , and \mathbb{C} denote the set of natural numbers, the set of real numbers, and the set of complex numbers respectively with the common calculation rules.

Problem 2

Let $p(z) = (z - i)(z + i)(z + 5)$, $A = p(D)$, and $b \in V$.

a) Is

$$Ax = b \tag{1}$$

a third order differential equation for $x \in V$?

b) Find the dimension of the null space of A .

c) Has equation (1) always a solution?

In Problem 3 you must use the previous problems to exemplify.

Problem 3

a) Do all vectors have a length?

b) Do all operators A have an adjoint operator A^* ?