V, W .... VEKTORROM

$T: V \to W$

$T(\vec{u}+\vec{v}) = T(\vec{u}) + T(\vec{v})$

$T(k\vec{u}) = kT(\vec{u})$

$T: V \to W$

LINEÆR TRANSFORMASJON

$\ker(T) = \{\vec{v} \in V \mid T(\vec{v}) = \vec{0}\}$

$T(\vec{0}) = \vec{0}$

$\ker(T)$

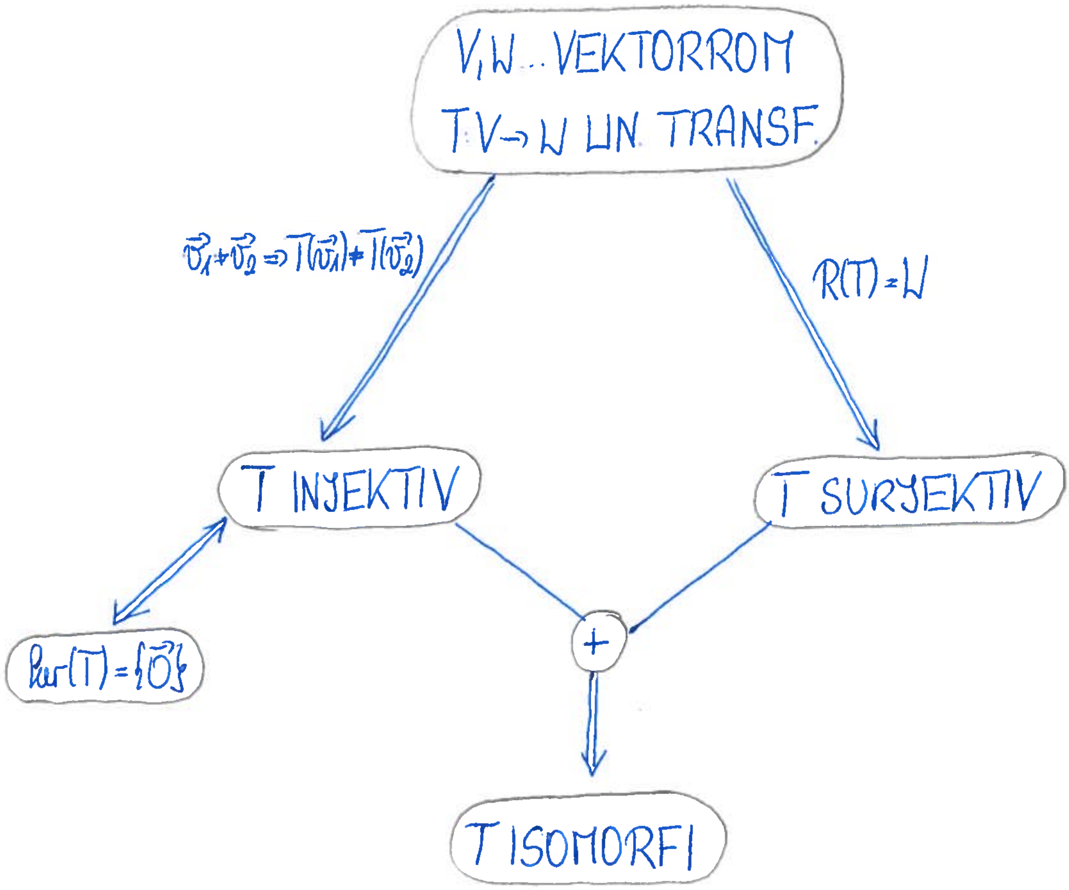
UNDERROM AV V

$R(T) = \{\vec{w} \in W \mid \exists \vec{v} \in V \text{ MED } T(\vec{v}) = \vec{w}\}$.

$T(\vec{0}) = \vec{0}$.

$R(T)$

UNDERROM AV W.


V,W ... VEKTORROM
T:V→W LIN. TRANSF.
$\vec{v}_1 \neq \vec{v}_2 \Rightarrow T(\vec{v}_1) \neq T(\vec{v}_2)$
R(T) = W
T INJEKTIV
T SURJEKTIV
Ker(T) = {$\vec{0}$}
+
T ISOMORFI

V... n-dim VEKTORROM
MED BASIS $\{\vec{v}_1, \vec{v}_2, \ldots, \vec{v}_n\}$
W... VEKTORROM
$T: V \to W$ LINEÆR TRANSF.

$\vec{v} = c_1\vec{v}_1 + c_2\vec{v}_2 + \ldots + c_n\vec{v}_n$

$T(\vec{v}) = c_1 T(\vec{v}_1) + c_2 T(\vec{v}_2) + \ldots + c_n T(\vec{v}_n)$

$\ker(T) = \{\vec{v} \in V \mid T(\vec{v}) = \vec{0}\}$

$\ker(T)$ .. UNDERROM AV V

$\dim(\ker(T)) \leq \dim(V) = n$

$\dim(\ker(T)) = \text{nullity}(T)$

$R(T) = \{\vec{w} \in W \mid \exists \vec{v} \in V$ MED $T(\vec{v}) = \vec{w}\}$

$R(T)$ UNDERROM AV W

$R(T) = \text{span}\{T(\vec{v}_1), T(\vec{v}_2), \ldots, T(\vec{v}_n)\}$.

$\dim(R(T)) = \text{rank}(T)$

$+$

n

$\ker(T) = \{\vec{0}\}$, $V = W$

T INJEKTIV $\Longleftrightarrow$ T SURJEKTIV

V ... n-dim VR MED BASIS $B = \{\vec{v}_1, \vec{v}_2, \dots, \vec{v}_n\}$

W ... m-dim VR MED BASIS $\tilde{B} = \{\vec{w}_1, \vec{w}_2, \dots, \vec{w}_m\}$.

$T: V \to W$ LIN TRANSF.

$$\begin{tikzcd}
V \arrow[r, "T"] \arrow[d, shift right, "T_1"'] & W \arrow[d, shift right, "T_2"'] \\
\mathbb{R}^n \arrow[r, "{[T]_{\tilde{B},B}}"'] \arrow[u, shift right, "T_1^{-1}"'] & \mathbb{R}^m \arrow[u, shift right, "T_2^{-1}"']
\end{tikzcd}
\qquad
\begin{tikzcd}
\vec{u} \arrow[r, "T"] \arrow[d, shift right, "T_1"'] & T(\vec{u}) \arrow[d, shift right, "T_2"'] \\
(\vec{u})_B \arrow[r, "{[T]_{\tilde{B},B}}"'] \arrow[u, shift right, "T_1^{-1}"'] & (T(\vec{u}))_{\tilde{B}} \arrow[u, shift right, "T_2^{-1}"']
\end{tikzcd}$$

$$T(\vec{u}) = T_2^{-1}\left([T]_{\tilde{B},B}(T_1(\vec{u}))\right) \quad \forall \vec{u} \in V$$

$T_1: V \to \mathbb{R}^n$ (ISOMORFI)

$\vec{u} = c_1\vec{v}_1 + c_2\vec{v}_2 + \dots + c_n\vec{v}_n \longmapsto (c_1, c_2, \dots, c_n) = (\vec{u})_B$

$T_1^{-1}: \mathbb{R}^n \to V$

$(\vec{u})_B = (c_1, c_2, \dots, c_n) \longmapsto c_1\vec{v}_1 + c_2\vec{v}_2 + \dots + c_n\vec{v}_n = \vec{u}$

$T_2: W \to \mathbb{R}^m$ (ISOMORFI)

$\vec{w} = k_1\vec{w}_1 + k_2\vec{w}_2 + \dots + k_m\vec{w}_m \longmapsto (k_1, k_2, \dots, k_m) = (\vec{w})_{\tilde{B}}$

$T_2^{-1}: \mathbb{R}^m \to W$

$(\vec{w})_{\tilde{B}} = (k_1, k_2, \dots, k_m) \longmapsto k_1\vec{w}_1 + k_2\vec{w}_2 + \dots + k_m\vec{w}_m = \vec{w}$

$[T]_{\tilde{B},B}$ ... MATRISEN TIL T RELATIV TIL B OG $\tilde{B}$
(n x m MATRISE)

$[T]_{\tilde{B},B} = \left((T(\vec{v}_1))_{\tilde{B}}, (T(\vec{v}_2))_{\tilde{B}}, \dots, (T(\vec{v}_n))_{\tilde{B}}\right)$

V n-dim VR MED BASISER $\mathcal{B}$ OG $\tilde{\mathcal{B}}$.
$T: V \to V$ LIN. OPERATOR.

$T = I$:

$T_1: V \to \mathbb{R}^n$
$\vec{u} \mapsto (\vec{u})_{\mathcal{B}}$
$T_2: V \to \mathbb{R}^n$
$\vec{u} \mapsto (\vec{u})_{\tilde{\mathcal{B}}}$
ER ISOMORFIER.

V OG $\mathbb{R}^n$ ER ISOMORFE

$T_1^{-1}: \mathbb{R}^n \to V$
$(\vec{u})_{\mathcal{B}} \mapsto \vec{u}$
$T_2^{-1}: \mathbb{R}^n \to V$
$(\vec{u})_{\tilde{\mathcal{B}}} \mapsto \vec{u}$

$$
\begin{tikzcd}
\vec{u} \arrow[r, "I"] \arrow[d, shift right, "T_1"'] & \vec{u} \arrow[d, shift right, "T_2"'] \\
(\vec{u})_{\mathcal{B}} \arrow[u, shift right, "T_1^{-1}"'] \arrow[r, "{[I]_{\tilde{\mathcal{B}},\mathcal{B}}}"] & (\vec{u})_{\tilde{\mathcal{B}}} \arrow[u, shift right, "T_2^{-1}"']
\end{tikzcd}
$$

$$[I]_{\tilde{\mathcal{B}},\mathcal{B}} = P_{\mathcal{B}\to\tilde{\mathcal{B}}}.$$

T inj:

$T: V \to V$
$\vec{u} \mapsto T(\vec{u})$
$T_1: V \to \mathbb{R}^n$
$\vec{u} \mapsto (\vec{u})_{\mathcal{B}}$
ER ISOMORFIER.

$$
\begin{tikzcd}
\vec{u} \arrow[r, "T"] \arrow[d, shift right, "T_1"'] & T(\vec{u}) \arrow[d, shift right, "T_1"'] \\
(\vec{u})_{\mathcal{B}} \arrow[u, shift right, "T_1^{-1}"'] \arrow[r, "{[T]_{\mathcal{B}}}"] & (T(\vec{u}))_{\mathcal{B}} \arrow[u, shift right, "T_1^{-1}"']
\end{tikzcd}
$$

$$[T]_{\mathcal{B}} = [T]_{\mathcal{B},\mathcal{B}}.$$

$$
\begin{tikzcd}
\vec{u} \arrow[r, "T"] \arrow[rr, bend left, "I"] \arrow[d, shift right, "T_1"'] & T(\vec{u}) \arrow[r, "T^{-1}"] \arrow[d, shift right, "T_1"'] & \vec{u} \arrow[d, shift right, "T_1"'] \\
(\vec{u})_{\mathcal{B}} \arrow[u, shift right, "T_1^{-1}"'] \arrow[r, "{[T]_{\mathcal{B}}}"] \arrow[rr, bend right] & (T(\vec{u}))_{\mathcal{B}} \arrow[u, shift right, "T_1^{-1}"'] \arrow[r, "{[T^{-1}]_{\mathcal{B}}}"] & (\vec{u})_{\mathcal{B}} \arrow[u, shift right, "T_1^{-1}"']
\end{tikzcd}
$$

$$[I]_{\mathcal{B}} = I$$

$$[T]_{\mathcal{B}}^{-1} = [T^{-1}]_{\mathcal{B}}.$$

$U, V, W \ldots$ VR

$T_1: U \to V$

$T_2: V \to W$ LIN. TRANSF.

$T_2 \circ T_1: U \longrightarrow W$

$\vec{u} \longmapsto T_2(T_1(\vec{u}))$ LIN. TRANSF

T_1 INJ.

T_2 INJ.

$T_2 \circ T_1$ INVERTERBAR

$(T_2 \circ T_1)^{-1}: R(T_2 \circ T_1) \longrightarrow U$

$\vec{w} \longmapsto T_1^{-1}(T_2^{-1}(\vec{w}))$

U, V, W ENDL-DIM

$\mathcal{B} \ldots$ BASIS TIL U

$\tilde{\mathcal{B}} \ldots$ BASIS TIL V

$\hat{\mathcal{B}} \ldots$ BASIS TIL W.

$[T_2 \circ T_1]_{\hat{\mathcal{B}}, \mathcal{B}} = [T_2]_{\hat{\mathcal{B}}, \tilde{\mathcal{B}}} [T_1]_{\tilde{\mathcal{B}}, \mathcal{B}}$.