1(a)

$$\begin{vmatrix} \lambda+1 & 0 & -1 \\ 3 & \lambda-2 & -1 \\ -12 & 0 & \lambda+2 \end{vmatrix} = (\lambda+1)(\lambda-2)(\lambda+2) - 12(\lambda-2) = (\lambda-2)\big((\lambda+1)(\lambda+2)-12\big)$$

$$= (\lambda-2)(\lambda^2+3\lambda-10)$$

$$= (\lambda-2)^2(\lambda+5)$$

$\Longrightarrow$ $\underline{\underline{\lambda = 2 \text{ og } \lambda = -5}}$

1(b)

$\lambda = 2$: $\begin{pmatrix} 3 & 0 & -1 \\ 3 & 0 & -1 \\ -12 & 0 & 4 \end{pmatrix} \sim \begin{pmatrix} 3 & 0 & -1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$

$\Longrightarrow$ basis for egenrom: $\left\{ \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \\ 3 \end{pmatrix} \right\}$

$\lambda = -5$: $\begin{pmatrix} -4 & 0 & -1 \\ 3 & -7 & -1 \\ -12 & 0 & -3 \end{pmatrix} \sim \begin{pmatrix} 1 & 0 & 1/4 \\ 3 & -7 & -1 \\ 0 & 0 & 0 \end{pmatrix} -3 \sim \begin{pmatrix} 1 & 0 & 1/4 \\ 0 & -7 & -7/4 \\ 0 & 0 & 0 \end{pmatrix} \sim \begin{pmatrix} 4 & 0 & 1 \\ 0 & 28 & 7 \\ 0 & 0 & 0 \end{pmatrix}$

$\Longrightarrow$ basis for egenrom: $\left\{ \begin{pmatrix} 1 \\ 1 \\ -4 \end{pmatrix} \right\}$

$$P = \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 0 & 3 & -4 \end{pmatrix}, \quad D = \begin{pmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & -5 \end{pmatrix}$$

2(a)

$$A \sim \begin{pmatrix} 3 & 5 & 20 & 30 \\ 9 & 15 & 30 & 75 \\ 3 & 5 & 10 & 25 \\ 3 & 5 & 10 & 33 \end{pmatrix} \begin{matrix} \\ \\ -3 \;\; -1 \\ \\ \end{matrix} \sim \begin{pmatrix} 0 & 0 & 10 & 5 \\ 0 & 0 & 0 & 0 \\ 3 & 5 & 10 & 25 \\ 0 & 0 & 0 & 8 \end{pmatrix} \sim \begin{pmatrix} 3 & 5 & 10 & 25 \\ 0 & 0 & 2 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$\Longrightarrow$ rank(A) = 3

nullity(A) = 1

2(b)

Ledende variabler i kolonne 1,3,4 (i den reduserte matrisen), så kolonne 1,3,4 i A danner basis for kol.rommet:

$$\mathcal{B} = \left\{ \begin{pmatrix} 3/5 \\ 9/5 \\ 3 \\ 1 \end{pmatrix}, \begin{pmatrix} 4 \\ 6 \\ 10 \\ 10/3 \end{pmatrix}, \begin{pmatrix} 6 \\ 15 \\ 25 \\ 11 \end{pmatrix} \right\}$$

3(a)

La $f_1(x) = a_1 + b_1x^2 + c_1x^4$ og $f_2(x) = a_2 + b_2x^2 + c_2x^4$ ligge i V, og $r \in \mathbb{R}$:

$$f_1(x) + f_2(x) = (a_1+a_2) + (b_1+b_2)x^2 + (c_1+c_2)x^4 \in V$$
$$r f_1(x) = (ra_1) + (rb_1)x^2 + (rc_1)x^4 \in V$$

La $g_1(x) = a_1 + b_1x + c_2x^3$ og $g_2(x) = c_2 + b_2x + c_2x^3$ ligge i W, og $r \in \mathbb{R}$:

$$g_1(x) + g_2(x) = (a_1+a_2) + (b_1+b_2)x + (c_1+c_2)x^3 \in W$$
$$r g_1(x) = (ra_1) + (rb_1)x + (rc_1)x^3 \in W$$

Siden V og W er lukket under addisjon og skalarmultiplikasjon, er de underrom av P_4.

B_1 er basis for V siden (1) span $B_1 = V$
(2) B_1 lin. uavh.

Rommet W har dimensjon 3, og B_2 genererer W. Siden $|B_2| = 3$ er da B_2 basis for W.

3(b)

To krav:

(1) $T(p(x)+q(x)) = \frac{d}{dx}(p(x)+q(x)) = p'(x)+q'(x) = T(p(x)) + T(q(x))$

(2) $T(rp(x)) = \frac{d}{dx}(rp(x)) = rp'(x) = rT(p(x))$

for alle $p(x), q(x) \in V$ og $r \in \mathbb{R}$. Derfor er T en lin. transf.

$$T(1) = 0 = 0\cdot(1+2x) + 0(x+x^3) + 0\cdot x^3 \implies [T(1)]_{B_2} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$T(x^2) = 2x = 0(1+2x) + 2(x+x^3) - 2x^3 \implies [T(x^2)]_{B_2} = \begin{pmatrix} 0 \\ 2 \\ -2 \end{pmatrix}$$

$$T(x^4) = 4x^3 = 0(1+2x) + 0(x+x^3) + 4x^3 \implies [T(x^4)]_{B_2} = \begin{pmatrix} 0 \\ 0 \\ 4 \end{pmatrix}$$

$$\implies [T]_{B_2,B_1} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & -2 & 4 \end{pmatrix}$$

3(c)

$\dim R(T) = \operatorname{rank}[T]_{B_2,B_1} = 2$.

Siden $\dim \operatorname{Ker}(T) = \dim V - \dim R(T) = 3-2 = 1$ er $\operatorname{Ker}(T) \neq \{0\}$, så T er <u>ikke</u> en-til-en.