# MA1201 Linear Algebra and Geometry

**Exercise set 06**

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## Compulsory exercises

Hand in your solutions to these exercises. All answers must be justified.

### Chapter 3.5 - Cross product

**Exercise 1** Do exercise 14 in chapter 3.5 of Elementary Linear Algebra.

The triangle is spanned by $A - C = (-2, 4)$ and $B - C = (-1, 5)$. The area of a triangle is half that of the associated parallellogram thus the area is

$$\left|\det\left(\begin{bmatrix}-2 & -1\\ 4 & 5\end{bmatrix}\right)\right| = |-6| = 6$$

**Exercise 2** Do exercise 22 in chapter 3.5 of Elementary Linear Algebra.

$$\mathbf{u}\cdot(\mathbf{v}\times\mathbf{w}) = \det\left(\begin{bmatrix}-1 & 3 & -1\\ 2 & 4 & 2\\ -4 & -2 & 5\end{bmatrix}\right) = -90$$

### Chapter 4.1 - Real vector spaces

Here are the vector space axioms as listed on page 203.

1. If $\mathbf{u}$ and $\mathbf{v}$ are in $V$, then $\mathbf{u}+\mathbf{v} \in V$.
2. $\mathbf{u}+\mathbf{v} = \mathbf{v}+\mathbf{u}$
3. $(\mathbf{u}+\mathbf{v})+\mathbf{w} = \mathbf{u}+(\mathbf{v}+\mathbf{w})$
4. There exists an object in $V$, called the zero vector, that is denoted by $\mathbf{0}$ and has the property that $\mathbf{0}+\mathbf{u} = \mathbf{u}+\mathbf{0} = \mathbf{u}$ for all $\mathbf{u}$ in $V$.
5. For each $\mathbf{u}$ in $V$, there is an object $-\mathbf{u}$ in $V$, called a negative of $\mathbf{u}$, such that $\mathbf{u}+(-\mathbf{u}) = (-\mathbf{u})+\mathbf{u} = \mathbf{0}$.
6. If $k$ is any scalar and $\mathbf{u} \in V$, then $k\mathbf{u} \in V$.

7. $k(\mathbf{u} + \mathbf{v}) = k\mathbf{u} + k\mathbf{v}$
8. $(k + m)\mathbf{u} = k\mathbf{u} + m\mathbf{u}$
9. $k(m\mathbf{u}) = (km)(\mathbf{u})$
10. $1\mathbf{u} = \mathbf{u}$

**Exercise 3** Do exercise 3-9 in chapter 4.1 of Elementary Linear Algebra.

3, 4, 6, and 9 are vector spaces. In exercise 5 axiom 5 and 6 fails. In exercise 7 axiom 8 fails. In exercise 8 axiom 1, 4, and 6 fails, and axiom 5 is illdefined without axiom 4.

## Chapter 4.2 - Subspaces

**Exercise 4** Do exercise 6 in chapter 4.2 of Elementary Linear Algebra.

(a) The polynomial $p(x) = 1 + x$ has rational coefficients, and $\sqrt{2}$ is a real scalar. But $\sqrt{2}p(x) = \sqrt{2} + \sqrt{2}x$ does not have rational coefficients. Hence this is not a subspace.

(b) Let $p(x) = a_0 + a_1 x$ and $q(x) = b_0 + b_1 x$ be polynomials of the relevant form, and let $k \in \mathbb{R}$ be a scalar. Then $p(x) + q(x) = (a_0 + b_0) + (a_1 + b_1)x$ and $kp(x) = ka_0 + ka_1 x$ are also of the relevant form. Hence, the set of such polynomials is a subspace.