Norwegian University of Science and Technology
Department of Mathematical Sciences

MA1201 Linear Algebra and Geometry

Exercise set 05

Compulsory exercises

Hand in your solutions to these exercises. All answers must be justified.

Chapter 2.1 - determinants and cofactors

Exercise 1 Exercise 3a in chapter 2.1 of Elementary Linear Algebra.

$$M_{13} = C_{13} = \det\left(\begin{bmatrix} 0 & 0 & 3 \\ 4 & 1 & 14 \\ 4 & 1 & 2 \end{bmatrix}\right) = 0$$

Exercise 2 Exercise 16 in chapter 2.1 of Elementary Linear Algebra.

$\det(A) = (\lambda - 4)(\lambda(\lambda - 1) - 2 \cdot 3) = (\lambda - 4)(\lambda - 3)(\lambda + 2)$. Which is 0 wehn λ is 4, 3 or -2.

Exercise 3 Exercise 22 in chapter 2.1 of Elementary Linear Algebra.

$$\begin{aligned} \det\left(\begin{bmatrix} 3 & 3 & 1 \\ 1 & 0 & -4 \\ 1 & -3 & 5 \end{bmatrix}\right) &= 3\det\left(\begin{bmatrix} 0 & -4 \\ -3 & 5 \end{bmatrix}\right) - 1\det\left(\begin{bmatrix} 3 & 1 \\ -3 & 5 \end{bmatrix}\right) + 1\det\left(\begin{bmatrix} 3 & 1 \\ 0 & -4 \end{bmatrix}\right) \\ &= 3(0 - 12) - (15 + 3) + (-12 - 0) \\ &= -66 \end{aligned}$$

Exercise 4 Exercise 29 in chapter 2.1 of Elementary Linear Algebra.

For a triangular matrix, the determinant is the product of the diagonal elements. Since there is a 0 along the diagonal the determinant is 0 in this case.

Chapter 2.2 - determinants and row reduction

Exercise 5 Exercise 10 in chapter 2.2 of Elementary Linear Algebra.

If we adde $\frac{2}{3}$ times the frist row to the third, and then swap the second and third row we get the following

$$\begin{bmatrix} 3 & 6 & -9 \\ 0 & 0 & -2 \\ -2 & 1 & 5 \end{bmatrix} \sim \begin{bmatrix} 3 & 6 & -9 \\ 0 & 0 & -2 \\ 0 & 5 & -1 \end{bmatrix} \sim \begin{bmatrix} 3 & 6 & -9 \\ 0 & 5 & -1 \\ 0 & 0 & -2 \end{bmatrix}$$

This reduced matrix has determinant $3 \cdot 5 \cdot (-2) = -30$, and swapping rows changes the sign of the determinant. Hence the original matrix has determinant 30.

Exercise 6 Exercise 16 in chapter 2.2 of Elementary Linear Algebra.

We see that we can obtain this matrix from the given one by swapping the first and third row. Hence the determiant is $-(-6) = 6$.

Exercise 7 Exercise 20 in chapter 2.2 of Elementary Linear Algebra.

We see that we can obtain this matrix from the given one by adding 3 times the first row to the third, and scaling the second by 2. Adding a row to another doesnt change the determinant, but scaling a row by 2 scales the determiannt by 2. Hence, the determiannt is $2 \cdot (-6) = -12$.

Chapter 2.3 - Cramer's rule

Exercise 8 Exercise 20 in chapter 2.3 of Elementary Linear Algebra.

The entries of A^{-1} are the cofactrs of A divided by the determinant of A.

$$A^{-1} = \frac{1}{\det(A)} \begin{bmatrix} -12 & 0 & -9 \\ -4 & -2 & -4 \\ 6 & 0 & 6 \end{bmatrix} = \frac{1}{-6} \begin{bmatrix} -12 & 0 & -9 \\ -4 & -2 & -4 \\ 6 & 0 & 6 \end{bmatrix} = \begin{bmatrix} 2 & 0 & 3/2 \\ 2/3 & 1/3 & 2/3 \\ -1 & 0 & -1 \end{bmatrix}$$